STABILITY OF STEEL BEAMS AND COLUMNS

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In accordance with Eurocodes and the UK National Annexes

L Gardner PhD CEng MICE MIStructE

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FOREWORD

This publication has been prepared by Dr Leroy Gardner of Imperial College London and updates earlier guidance given in the SCI publication *Lateral stability of steel beams and columns – common cases of restraint* (P093) written by Professors David Nethercot and Mark Lawson and published in 1992. The present publication updates the guidance in line with the Eurocodes and responds to comments from SCI members on the advice given in the earlier publication.

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SUMMARY

This publication provides guidance on the determination of buckling resistance of beams and columns in accordance with Eurocode 3. The theory of elastic stability of beams and columns is reviewed briefly and the requirements of the Eurocode are explained. The typical forms of end restraint and intermediate restraint, which influence the buckling resistance, are illustrated and their influence discussed. The recommendations made are consistent with BS EN 1993-1-1, the UK National Annex and relevant non-contradictory complementary information (NCCI). Four simple worked examples are included.

NOTATION

a	Distance between restrained longitudinal axis and shear centre of member
b	Width of cross-section
b_q	Spacing of main beams in U-frame
b_r	Overall length of sheeting perpendicular to beam
b_s	Beam spacing
c	Taper factor; notch length; U-frame stiffness per unit length
d_c	Notch depth
e_0	Imperfection amplitude
f	Flange width covered by precast slab
f_y	Yield strength
g	Curvature parameter; gap between precast concrete slabs
h	Depth of cross-section
h_e	Exposed section height
h_f	Distance between shear centres of beam flanges
h_v	Distance between centroid of compression flange and top of cross-member in U-frame
h_w	Depth of profiled sheeting
i	Radius of gyration
i_s	Polar radius of gyration about restrained longitudinal axis
k	Effective length parameter
k_T	Restraint stiffness per unit length
k_w	Effective length parameter, accounting for warping restraint
l	U-frame spacing
m	Number of members to be restrained; number of half-waves into which a column buckles
n_b	Number of half-waves within a length for which M_{crT} is a minimum
q_d	Design value of equivalent stabilizing force
r_n	Reduction factor on M_{cr} for notched beam
s	Spacing between intermediate tension flange restraints; spacing of member supporting sheeting
t_f	Flange thickness
t_w	Web thickness
w	Uniformly distributed load per unit area
w_l	Uniformly distributed load per unit length
x	Distance to which slab covers beyond centreline of steel section
z_g	Distance between level of load application and shear centre of beam

A	Cross-sectional area
A_{eff}	Effective cross-sectional area
C_1	Equivalent uniform moment factor
C_2	Load level parameter
C_{d}	Stiffness of a discrete U-frame
C_{n}	Modification factor for nonlinear moment gradients
C_{m}	Modification factor for linear moment gradients
$C_{\theta,\mathrm{k}}$	Rotation restraint stiffness per unit length of beam
D	Destabilising parameter
E	Modulus of elasticity
F	Restraint force
G	Shear modulus
I_{q}	Second moment of area of cross-member in U-frame
I_{T}	Torsion constant
I_{v}	Second moment of area of vertical stiffeners in U-frame
I_{w}	Warping constant
I_{y}	Major axis second moment of area
I_{z}	Minor axis second moment of area
K_{T}	Restraint stiffness
L	Length/ system length
L_{cr}	Buckling length/ effective length
L_{k}	Stable length between torsional restraints under uniform moment
L_{m}	Stable length between lateral restraints
L_{s}	Stable length between torsional restraints under non-uniform moment
L_{stable}	Stable length between compression flange restraints
L_{t}	Length of member between effective torsional restraints
M_{cr}	Elastic critical moment for lateral torsional buckling
$M_{\mathrm{cr,n}}$	Elastic critical moment for lateral torsional buckling of notched beam
$M_{\mathrm{cr,T}}$	Elastic critical moment for torsional buckling about restrained axis
M_{Ed}	Design bending moment
M_{Res}	Restoring moment per unit length
$N_{\mathrm{b,Rd}}$	Design buckling resistance of column
N_{cr}	Elastic buckling load/ critical force (Euler load)
N_{crit}	Elastic critical force employed in U-frame design
$N_{\mathrm{cr,T}}$	Elastic critical force for torsional buckling about restrained axis
N_{E}	Euler load of unrestrained column
N_{Ed}	Design value of axial force
$N_{\mathrm{f,Ed}}$	Design value of axial force in compression flange at plastic hinge location
N_{y}	Yield load
Q_{m}	Local force applied at plastic hinge locations
S	Shear stiffness of sheeting per unit beam length
U	Parameter dependent on section geometry
V	Parameter related to slenderness
W_{el}	Elastic section modulus
W_{pl}	Plastic section modulus
W_{y}	Major axis section modulus
X	Torsional index

α	Imperfection factor for columns; factor depending on notch geometry
α_{cr}	Elastic buckling load factor of frame in sway mode
α_{LT}	Imperfection factor for beams
α_{m}	Reduction factor on imperfection when multiple beams are being restrained
β	Parameter that influences shape of buckling curves for beams
β_{w}	Factor that allows for the classification of a cross-section
γ_{M0}	Partial factor for cross-section resistance
γ_{M1}	Partial factor for member buckling resistance
δ_{q}	Deflection of bracing system
ε	$= \sqrt{235 / f_{\mathrm{y}}}$
η	Column imperfection parameter; Non-dimensional load level parameter
η_{LT}	Beam imperfection parameter
κ	Torsion parameter
λ	Geometric slenderness ratio L/i
λ_1	Limiting slenderness (at which $\sigma_{\mathrm{cr}} = f_{\mathrm{y}}$)
$\bar{\lambda}$	Non-dimensional column slenderness
$\bar{\lambda}_0$	Plateau length of column buckling curves
$\bar{\lambda}_{\mathrm{LT}}$	Non-dimensional beam slenderness
$\bar{\lambda}_{\mathrm{LT,0}}$	Plateau length of lateral torsional buckling curves
μ	Coefficient of friction
σ_{cr}	Elastic buckling stress
χ	Column buckling reduction factor
χ_{LT}	Lateral torsional buckling reduction factor
$\chi_{\mathrm{LT,mod}}$	Modified lateral torsional buckling reduction factor
ψ	Ratio of end moments
Φ	Intermediate factor in determining χ
Φ_{LT}	Intermediate factor in determining χ_{LT}

INTRODUCTION

The stability of beams and columns and the determination of their buckling resistances is an integral part of the design of steel framed buildings, both single storey and multi-storey. However, many designers experience difficulty in evaluating the effects of the many different types of end and intermediate restraints to beams and columns and thus in determining the buckling resistance. The difficulty often arises because practical restraint situations do not conform exactly to the standardized situations given in application rules. This publication provides guidance, with particular reference to design to the Eurocodes, that will assist designers to make safe yet not overly conservative evaluations of buckling resistance without the need to resort to complex analysis.

1.1 Design to the Eurocodes

The current standard for the design of steel framed buildings in the UK is Eurocode 3; the design of beams and columns is covered principally by BS EN 1993-1-1 [1] and its accompanying National Annex. The standard gives principles and application rules for the design of steel members, including rules for determining buckling resistance. General guidance on design in accordance with the Eurocodes is given in a number of publications, including *Introduction to the Eurocodes* [2] and *Medium rise braced frames* [3]. The guidance in the present publication complements the rules in the Eurocodes and in other SCI publications.

For brevity in this publication, references to clauses in BS EN 1993-1-1 simply quote the clause, figure or expression number – for example 6.3.1 would refer to Clause 6.3.1 and (6.46) would refer to Expression (6.46). Where other Eurocode Parts are referenced, a full reference is given.

1.2 Scope of this publication

This publication is concerned principally with the lateral buckling of columns and the lateral torsional buckling of beams. The theoretical aspects of buckling in its various forms are not covered in great detail, since there are existing comprehensive texts on this subject.

Section 2 addresses the rules in BS EN 1993-1-1 for the buckling resistance of members in buildings. It does not address the resistance of cross sections, including the effect of local buckling of elements of the cross section. Guidance is given on the use of the rules for the different types of restraint to beams and columns. As well as references to BS EN 1993-1-1, Section 2 refers to several sources of non-contradictory complementary information (NCCI) that facilitate use of the Eurocode rules; most of these documents may be found at the website www.steel-ncci.co.uk.

Section 3 describes in more detail the practical forms of restraints to beams and indicates how values for various parameters used in Section 2 may be chosen to suit particular situations. Section 4 discusses the treatment of stabilizing and destabilizing loads on beams and Section 5 covers the additional demands on bracing systems when frames are designed plastically. Section 6 covers the practical forms of restraint to columns and shows how these forms affect the calculation of buckling resistance. Four simple worked examples are included.

THEORETICAL BACKGROUND

The starting point for the stability design of structural components is elastic buckling theory. The most fundamental case is that of an axially compressed column whose buckling load is given by the well known Euler expression (Equation 2.1), though the general form of the Euler expression also features in many other stability problems. Consideration of theoretical buckling solutions is particularly relevant when designing to Eurocode 3 since elastic buckling loads and moments (referred to in Eurocode 3 as elastic critical forces and moments) feature far more prominently in the design calculations than in previous British Standards.

2.1 Column buckling

2.1.1 Elastic critical force

The elastic critical force for flexural buckling of a perfect, pin-ended column (i.e. the Euler load) is given by Equation (2.1):

$$N_{\mathrm{cr}} = \frac{\pi^2 EI}{L^2} \qquad \text{(for pin-ended columns)} \tag{2.1}$$

where

I is the second moment of area of the section

E is the modulus of elasticity (taken as 210000 N/mm^2 for steel)

L is the length of the column.

For end conditions other than pinned, the elastic critical force may be obtained from Equation (2.2), where the actual column length L has been replaced by its buckling length L_{cr} $(= kL)$, often referred to as its effective length. For a pin-ended column, the buckling length $L_{\mathrm{cr}} = L$, while other end conditions are discussed throughout this guide.

$$N_{\mathrm{cr}} = \frac{\pi^2 EI}{L_{\mathrm{cr}}^2} \qquad \text{(for columns with any end conditions)} \tag{2.2}$$

Below the theoretical buckling load N_{cr}, there are no lateral deflections, but upon reaching N_{cr}, lateral deflections grow rapidly. Writing N_{cr} in terms of stress σ_{cr} by dividing through by the cross-sectional area A, and defining the radius of gyration i from $I = Ai^2$, gives

$$\sigma_{\mathrm{cr}} = \frac{N_{\mathrm{cr}}}{A} = \frac{\pi^2 E(Ai^2)}{L_{\mathrm{cr}}^2 A} = \frac{\pi^2 E}{(L_{\mathrm{cr}}/i)^2} = \frac{\pi^2 E}{\lambda^2} \tag{2.3}$$

in which λ is the geometric slenderness ratio (L_{cr}/i) of the column. In Clause 6.3.1 of BS EN 1993-1-1, the parameter 'non-dimensional slenderness' $\bar{\lambda}$, is introduced:

$$\bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} \tag{2.4}$$

The parameter $\bar{\lambda}$ is a measure of slenderness that depends on both the material properties (E and f_y) and the geometric properties of the member. The non-dimensional slenderness $\bar{\lambda}$ is simply the geometric slenderness L_{cr}/i normalized by a limiting slenderness λ_1 at which the elastic buckling stress σ_{cr} of the column is equal to its yield strength f_y, as given by Equation (2.5):

$$\bar{\lambda} = \frac{L_{cr}/i}{\lambda_1} \tag{2.5}$$

The limiting slenderness λ_1 (where $\sigma_{cr} = f_y$) may be found to be:

$$\lambda_1 = \pi\sqrt{\frac{E}{f_y}} \tag{2.6}$$

Minor manipulation reveals that Equations (2.5) and (2.6) are indeed equivalent to Equation (2.4), as demonstrated in Equation (2.7).

$$\bar{\lambda} = \frac{L_{cr}/i}{\lambda_1} = \frac{\pi\sqrt{\dfrac{E}{\sigma_{cr}}}}{\pi\sqrt{\dfrac{E}{f_y}}} = \frac{\sqrt{\dfrac{1}{\sigma_{cr}}}}{\sqrt{\dfrac{1}{f_y}}} = \sqrt{\frac{f_y}{\sigma_{cr}}} = \sqrt{\frac{Af_y}{N_{cr}}} \tag{2.7}$$

Values of λ_1 for different steel grades (i.e. varying yield strengths) are given in Table 2.1, and may be used to determine $\bar{\lambda}$ directly from the slenderness ratio λ through Equation (2.5).

Table 2.1
Values of λ_1 for different steel grades

STEEL GRADE	THICKNESS RANGE (mm)	YIELD STRENGTH f_y (N/mm²)	λ_1
S275	$t \leq 16$	275	86.8
S275	$16 < t \leq 40$	265	88.4
S355	$t \leq 16$	355	76.4
S355	$16 < t \leq 40$	345	77.5

The approach of defining a component’s slenderness by means of its yield and elastic buckling characteristics is common throughout Eurocode 3.

Columns usually have different second moments of area about different buckling axes (e.g. I sections). Major and minor axis flexural buckling of an I section member typically relate to flexural deformations in the directions parallel and perpendicular to the web respectively. Note that the axes convention in Eurocode 3 is such that the y-y axis is the major axis and the z-z axis is the minor axis, as shown in Figure 2.1. According to Euler theory, lateral buckling will occur about the z-z axis (i.e. about the weaker axis) if $i_z < i_y$ unless lateral displacement is restrained in this direction.

In addition to the familiar flexural buckling mode described above, struts may also buckle by either pure twisting about their longitudinal axis or a combination of bending and twisting. The first type of behaviour is referred to as torsional buckling and is only possible for centrally-loaded doubly-symmetrical cross sections for which the centroid and shear centre coincide. The second, rather more general, form of response is termed torsional-flexural buckling and can occur in struts such as channels for which the centroid and shear centre do not coincide. The elastic critical forces for torsional buckling $N_{\mathrm{cr,T}}$ and for torsional-flexural $N_{\mathrm{cr,TF}}$ buckling are given in BS EN 1993-1-3[4], since these forms of buckling are most prevalent in cold-formed sections. A member will buckle in the mode (flexural, torsional or torsional-flexural) that has the lowest elastic critical force; for hot-rolled steel sections, this is generally the flexural buckling mode.

2.1.2 Flexural buckling resistance

The above discussion relates to the elastic buckling of perfect columns. Real columns or other compression members (often termed struts) behave in a different manner. The presence of an initial lack of straightness and/or small eccentricities of loading will mean that the column or strut will develop lateral deformations gradually rather than as a sudden process. Thus yielding will develop from the more heavily compressed regions, leading to a progressive loss of stiffness. This in turn will be influenced by the presence of residual stresses. Since the actual magnitude and distribution of factors like initial bow, residual stress, etc. will vary both between section types and, to some extent, within different samples of the same section, the actual relationship between column strength and slenderness will spread over a relatively wide band.

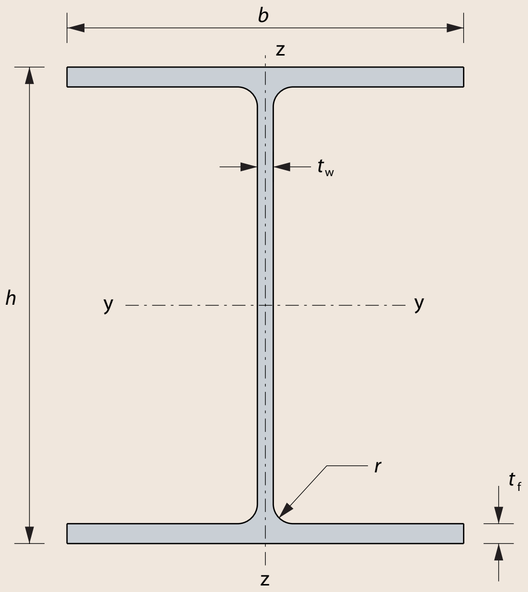


Figure 2.1 Labelling and axes convention in Eurocode 3

Fortunately there is some pattern to this spread. Clause 6.3.1.2 recognises this by providing five buckling curves, (see Figure 2.2), each of which can be represented by a modified Perry-Robertson formula, as follows:

$$(\sigma_{\mathrm{cr}} - \chi f_{\mathrm{y}})(f_{\mathrm{y}} - \chi f_{\mathrm{y}}) = \eta \sigma_{\mathrm{cr}} \chi f_{\mathrm{y}} \tag{2.8}$$

where

χ is the buckling reduction factor

Note that the product of χ and f_{y} is equivalent to the compressive strength of the column, denoted p_{c} in BS 5950-1[5].

η $= \alpha(\bar{\lambda} - \bar{\lambda}_0)$

$\bar{\lambda}_0$ $= 0.2$

α is an imperfection factor, according to the buckling curve.

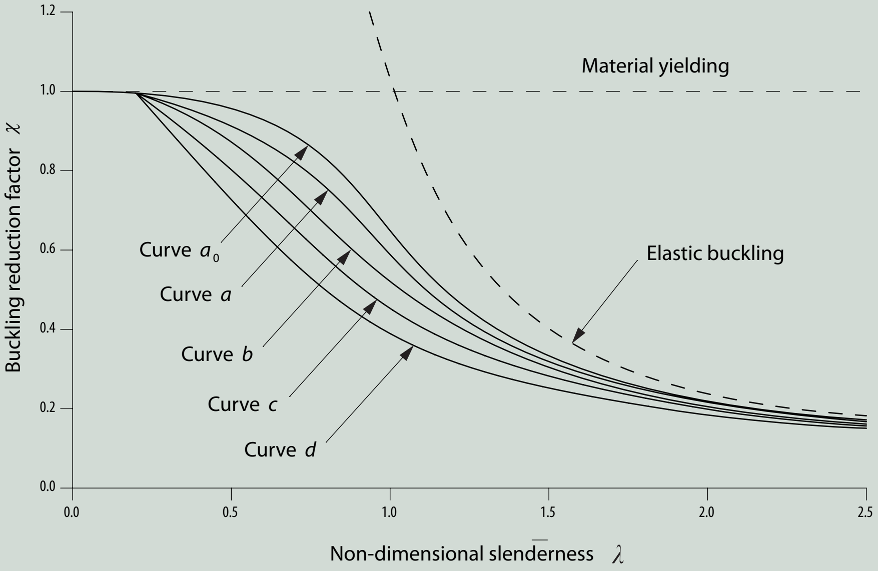


Figure 2.2 Column buckling curves in BS EN 1993-1-1

The coefficient η is an imperfection parameter dependent on the type of section and the method of forming (i.e. hot-finished, cold-formed or welded), and is related to the slenderness of the column. The original Perry formula (without $\bar{\lambda}_0$) was based on the onset of yielding at any point in the cross-section of the column. Inclusion of a limiting slenderness $\bar{\lambda}_0$ (below which column buckling is insignificant) and values of the imperfection factor α determined from tests allows for actual column failure (not necessarily first yield).

Table 6.1 of BS EN 1993-1-1 defines the five design curves (a_0, a, b, c and d) by their values for the imperfection factor α (0.13, 0.21, 0.34, 0.49 and 0.76). Solving Equation (2.8) (lower root) gives:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \quad \text{but } \chi \leq 1 \tag{2.9}$$

in which Φ is given in Clause 6.3.1.2 as:

$$\Phi = 0.5[1 + \eta + \bar{\lambda}^2] = 0.5[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2] \tag{2.10}$$

From the expression for η, it follows that the buckling reduction factor $\chi = 1.0$ when $\bar{\lambda} = \bar{\lambda}_0$, meaning that below this limiting slenderness the column is sufficiently stocky that no significant reduction in cross-section resistance occurs. In such a case, the characteristic value of the compressive resistance of the column is given by the product Af_y, provided the cross-section itself is not Class 4.

For the general case, the design buckling resistance of a compression member $N_{b,Rd}$ is defined in Clause 6.3.1.1 as:

$$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}} \quad \text{(for Class 1, 2 and 3 cross-sections)} \tag{2.11}$$

where χ is determined, for the relevant buckling mode and buckling curve, from Equation (2.9) and γ_{M1} is the partial safety factor for member buckling ($= 1.0$ according to NA.2.15).

Selection of the appropriate curve is made with reference to Table 6.2 of BS EN 1993-1-1. Essentially, sections with low imperfections and low residual stress, which perform well as columns, are designed using a high curve, while sections that perform less well are designed using one of the lower curves. For the more commonly used section types and steel grades (excluding S460), the requirements of BS EN 1993-1-1 in terms of buckling curve and corresponding imperfection factor α (from Table 6.1 of BS EN 1993-1-1) are summarised in Table 2.2. For buckling curves for grade S460, refer to Table 6.2 of BS EN 1993-1-1.

For angles, $\alpha = 0.34$ (curve b) and for channels, $\alpha = 0.49$ (curve c). For welded sections, curve b, c or d is applicable depending on material thickness, weld size, axis of buckling and material strength (see Table 6.2 of BS EN 1993-1-1).

Table 2.2 Column buckling curves & imperfection factors α, based on Tables 6.1 and 6.2 of BS EN 1993-1-1

SECTION TYPE	THICKNESS RANGE (mm)	BUCKLING AXIS	BUCKLING CURVE (UP TO S420)	IMPERFECTION FACTOR α
UKC	$t_f \leq 100$	Major (y-y)	b	0.34
		Minor (z-z)	c	0.49
UKB	$t_f \leq 40$	Major (y-y)	a	0.21
		Minor (z-z)	b	0.34
	$40 < t_f \leq 100$	Major (y-y)	b	0.34
		Minor (z-z)	c	0.49
Hot finished hollow section	Any	Any	a	0.21

The influence of varying end conditions and lateral restraints may be accounted for through the effective length concept. The buckling length of a column $L_{\mathrm{cr}} = kL$, where L is the length of the column between points of effective lateral restraint and k is the effective length parameter, values of which are provided in this guide.

For flexural buckling, the Eurocode 3 column design process therefore consists of the following steps:

- Select trial section
- Determine the buckling length of the column L_{cr} in the y and z directions
- Calculate $\bar{\lambda} = (L_{\mathrm{cr}} / i)/\lambda_1$ (equivalent to $\bar{\lambda} = \sqrt{Af_{\mathrm{y}} / N_{\mathrm{cr}}}$) in the y and z directions (using the appropriate value of i)
- Select appropriate buckling curve (a_0, a, b, c, or d) and corresponding imperfection factor α
- Obtain buckling reduction factor χ for each direction and select lower value
- Calculate $N_{\mathrm{b,Rd}} = \chi A f_{\mathrm{y}}/\gamma_{\mathrm{M1}}$ and verify adequacy for resisting the design compression force.

For checking torsional and flexural torsional buckling, the process is the same, except that non-dimensional slenderness should be obtained from Equation (2.4) on the basis of the elastic critical force for each mode. Further guidance and design examples are given in other publications, such as Designers’ Guide to EN 1993-1-1 [6].

2.2 Beam buckling

2.2.1 Elastic critical moment

Beams of open section bent in their stiffer principal plane are susceptible to a type of buckling involving a combination of lateral deflection and twist as illustrated in Figure 2.3. This is known as lateral torsional buckling. An approach similar to that for the Euler buckling of struts may be used to determine the elastic critical moment M_{cr}. For the idealized case of loading and support, taken to be uniform single curvature bending and beam ends that cannot deflect vertically or laterally or twist (but are provided with no other restraining effects) for a beam that is symmetric about the major axis, the expression for the elastic critical moment M_{cr} is obtained as:

$$M_{\mathrm{cr}} = \frac{\pi}{L}\sqrt{EI_{\mathrm{z}}GI_{\mathrm{T}}}\sqrt{1+\frac{\pi^2 EI_{\mathrm{w}}}{L^2 GI_{\mathrm{T}}}} \tag{2.12}$$

where

E is the modulus of elasticity

I_{z} is the minor (z-z) axis second moment of area of the section

G is the shear modulus

I_{T} is the torsion constant of the section

I_{w} is the warping constant of the section

L is the length of the beam.

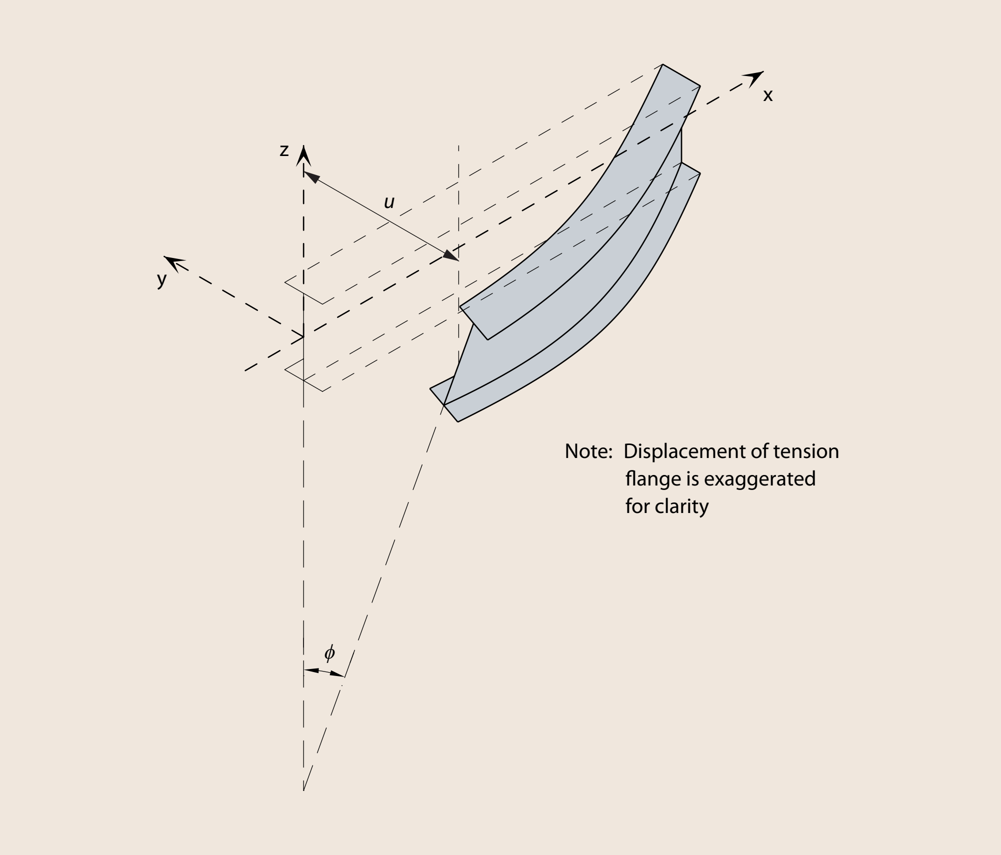

x
z
u
y
ϕ
Note: Displacement of tension flange is exaggerated for clarity

Figure 2.3
Deformations u and ϕ associated with lateral torsional buckling

For non-uniform bending and differing degrees of end restraint against rotation on plan, this expression is extended to:

$$M_{\mathrm{cr}} = C_1 \frac{\pi}{kL}\sqrt{EI_{\mathrm{z}}GI_{\mathrm{T}}}\sqrt{1+\frac{\pi^2 EI_{\mathrm{w}}}{(kL)^2 GI_{\mathrm{T}}}} \tag{2.13}$$

where

C_1 is the equivalent uniform moment factor and accounts for the shape of the bending moment diagram, as discussed in Section 2.4.

k is an effective length parameter, values of which are given in Section 3 of this guide.

A more general expression that allows for the shape of the bending moment diagram, different end restraint conditions, warping restraints, initial curvature and the level at which the load is applied is given in NCCI SN002[7] as:

$$M_{\mathrm{cr}} = C_1 \frac{\pi^2 EI_{\mathrm{z}}}{(kL)^2 g}\left(\sqrt{\left(\frac{k}{k_{\mathrm{w}}}\right)^2 \frac{I_{\mathrm{w}}}{I_{\mathrm{z}}} + \frac{(kL)^2 GI_{\mathrm{T}}}{\pi^2 EI_{\mathrm{z}}} + (C_2 z_{\mathrm{g}})^2} - C_2 z_{\mathrm{g}}\right) \tag{2.14}$$

where

g allows for in-plane curvature of the beam prior to buckling and is given by $g = \sqrt{\left(1-\frac{I_{\mathrm{z}}}{I_{\mathrm{y}}}\right)}$ or may conservatively be taken as unity.

k_w is a warping restraint parameter; where no warping restraint is provided, and as a conservative assumption when the degree of warping restraint is uncertain, k_w should be taken as unity

z_g is the distance between the level of application of the loading and the shear centre; z_g is positive for destabilizing loads applied above the shear centre

C_2 is a parameter associated with the load level and is dependent on the shape of the bending moment diagram (see Section 2.5 and NCCI SN003 [8]).

A useful tool for the determination of the elastic buckling moment M_{cr} for numerous section types, loading configurations and support conditions is also available in the form of the freely downloadable software *LTBeam* [9]. Practical advice on the use of this software has also been published [10].

2.2.2 Lateral torsional buckling resistance

The relationship between the elastic critical buckling moment M_{cr} and the design buckling resistance moment $M_{b,Rd}$ is defined in Clause 6.3.2.1 as:

$$M_{b,Rd} = \frac{\chi_{LT} W_y f_y}{\gamma_{M1}} \qquad (2.15)$$

where

χ_{LT} is the reduction factor for lateral torsional buckling (Equation (2.16))

W_y is the major axis section modulus – $W_{pl,y}$ (plastic section modulus) for Class 1 and 2 sections, $W_{el,y}$ (elastic section modulus) for Class 3 sections and $W_{eff,y}$ (effective section modulus) for Class 4 sections

γ_{M1} is the partial safety factor for member buckling, taken as 1.0 for buildings. The relationship is similar to that between N_{cr} and $N_{b,Rd}$ for a column.

For beams, there is a choice between two sets of lateral torsional buckling curves – these are set out in Clause 6.3.2.2 for the 'general case', which may be applied to any section type, and Clause 6.3.2.3 for 'rolled or equivalent welded sections' (equivalent welded sections being those of similar size and proportions to standard rolled sections).

For the general case, the buckling curves are given in Table 6.4 of BS EN 1993-1-1 and the reduction factor for lateral torsional buckling is given as:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \quad \text{but } \chi_{LT} \le 1 \qquad (2.16)$$

in which

$$\Phi_{LT} = 0.5[1 + \eta_{LT} + \bar{\lambda}_{LT}^2] = 0.5[1 + \alpha_{LT}(\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2] \qquad (2.17)$$

where α_{LT} is the imperfection factor given by Table 6.3 and $\bar{\lambda}_{LT}$ is the non-dimensional slenderness of the beam, defined, analogously to columns, by the square root of the ratio of its cross-section major (y-y) axis bending resistance $W_y f_y$ to its elastic buckling moment M_{cr}, as given by Equation (2.18).

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} \tag{2.18}$$

Although the 'default' plateau length (i.e. the non-dimensional slenderness below which the full in-plane bending moment resistance may be achieved) is set at 0.2 in Clause 6.3.2.2(1), a concession that allows lateral torsional buckling effects to be ignored up to a slenderness of $\bar{\lambda}_{LT,0}$ is given in Clause 6.3.2.2(4). $\bar{\lambda}_{LT,0}$ is defined in Clause 6.3.2.3 and its value is given in NA.2.17 as 0.4 for all rolled sections, including hollow sections; this generates a step in the buckling curves at this value. For welded sections, $\bar{\lambda}_{LT,0}$ is set at 0.2 in NA.2.17.

For rolled or equivalent welded sections, the modified buckling curves (Equations (2.19) and (2.20)) given in Clause 6.3.2.3 of BS EN 1993-1-1 may be applied. Reference is now made to Table 6.5 of BS EN 1993-1-1 for the selection of buckling curves for different section types, but note that this table is replaced in NA.2.17. The reduction factor for lateral torsional buckling is given in Clause 6.3.2.3 as:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta\bar{\lambda}_{LT}^2}} \quad \text{but} \begin{cases} \chi_{LT} \le 1.0 \\ \chi_{LT} \le \dfrac{1}{\bar{\lambda}_{LT}^2} \end{cases} \tag{2.19}$$

in which

$$\Phi_{LT} = 0.5[1 + \eta_{LT} + \beta\bar{\lambda}_{LT}^2] = 0.5[1 + \alpha_{LT}(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta\bar{\lambda}_{LT}^2] \tag{2.20}$$

The values of $\bar{\lambda}_{LT,0}$ and β are defined in NA.2.17, with $\bar{\lambda}_{LT,0}$ being as described above, while β is set at 0.75 for all rolled sections and 1.0 for welded sections. Generally, more favourable results are derived from Clause 6.3.2.3 than Clause 6.3.2.2, but owing to the different choice of buckling curves for the two cases (Table 6.5 as modified by the UK National Annex and Table 6.4, respectively), this is not always true[11].

An additional source of enhanced economy for rolled or equivalent welded sections is available through the f factor, which may be used to derive a modified buckling reduction factor $\chi_{LT,mod}$, as described in Clause 6.3.2.3 of BS EN 1993-1-1. Adopting $\chi_{LT,mod}$ is always beneficial, so it could be safely ignored. Further analysis and discussion of the choice of lateral torsional buckling curves is given in Reference 11.

2.2.3 Simplified assessment of buckling resistance

A simplified design method for assessing the lateral stability of beams in buildings with discrete restraints to the compression flange is given in Clause 6.3.2.4. In this method, the lateral buckling response of the compression flange of the beam plus one third of the compressed portion of the web, analysed as a strut, is assumed to represent the lateral torsional buckling behaviour of the beam. Further discussion of the method is given in Reference 6.

2.3 Simplified determination of slenderness

The basic definition of non-dimensional beam slenderness $\bar{\lambda}_{\mathrm{LT}}$ (Equation (2.18)) requires the explicit calculation of M_{cr}, given, for the most general case, by Equation (2.14). Use of this equation will generally lead to the most accurate assessment of lateral torsional buckling resistance and hence the most economic design. There are, however, a number of simplifications that can be made in the determination of $\bar{\lambda}_{\mathrm{LT}}$ that will greatly expedite the calculation process, often with little loss of economy. These simplifications are described in NCCI SN002[7] and are summarised below. A number of the simplifications relate specifically to doubly-symmetric I sections.

As an alternative to Equation (2.18), the non-dimensional beam slenderness $\bar{\lambda}_{\mathrm{LT}}$ may be taken as:

$$\bar{\lambda}_{\mathrm{LT}} = \frac{1}{\sqrt{C_1}} UVD\bar{\lambda}_{\mathrm{z}}\sqrt{\beta_{\mathrm{w}}} \qquad (2.21)$$

C_1 is a factor that allows for the shape of the bending moment diagram and is discussed in Section 2.4. It may be conservatively taken as equal to 1.0.

U is a parameter that depends on section geometry, given by:

$$U = \sqrt{\frac{W_{\mathrm{pl,y}}g}{A}\sqrt{\frac{I_{\mathrm{z}}}{I_{\mathrm{w}}}}} \qquad (2.22)$$

where all symbols are as previously defined.

For UKB and UKC sections, values of U range between about 0.84 and 0.90; $U = 0.9$ is therefore a suitable conservative upper bound for such sections. The parameter g is defined in Section 2.2.1.

V is a parameter related to slenderness, and is given in full by:

$$V = \frac{1}{\sqrt[4]{\left(\frac{k}{k_{\mathrm{w}}}\right)^2 + \frac{\lambda_{\mathrm{z}}^2}{\frac{\pi^2 E}{G}\frac{A}{I_{\mathrm{T}}}\frac{I_{\mathrm{w}}}{I_{\mathrm{z}}}} + (C_2 z_{\mathrm{g}})^2\frac{I_{\mathrm{z}}}{I_{\mathrm{w}}}}} \qquad (2.23)$$

the symbols for which are defined in Section 2.2.1.
For doubly-symmetric hot-rolled UKB and UKC sections, and for cases where the loading is not destabilizing, V may be conservatively simplified to:

$$V = \frac{1}{\sqrt[4]{1 + \frac{1}{20}\left(\frac{\lambda_{\mathrm{z}}}{h/t_{\mathrm{f}}}\right)^2}} \qquad (2.24)$$

For all sections symmetric about the major axis and not subjected to destabilizing loading, V may be conservatively taken as equal to 1.0.

D is a destabilizing parameter to allow for destabilizing loads (i.e. loads applied above the shear centre of the beam, where the load can move with the beam as it buckles), given by:

$$D = \frac{1}{\sqrt{1 - V^2 C_2 z_g \sqrt{\frac{I_z}{I_w}}}} \tag{2.25}$$

Destabilizing loads are discussed in Section 2.5. For non-destabilizing loads, $D = 1.0$.

$\bar{\lambda}_z$ is the minor axis non-dimensional slenderness of the member, given by $\bar{\lambda}_z = \lambda_z / \lambda_1$, in which $\lambda_z = kL / i_z$, where k is an effective length parameter, values of which are given in Section 3 of this guide.

β_w is a parameter that allows for the classification of the cross-section; for Class 1 and 2 sections, $\beta_w = 1$ while for Class 3 sections $\beta_w = W_{el,y} / W_{pl,y}$.

For a hot-rolled doubly-symmetric I or H section with lateral restraints to the compression flange at both ends of the segment under consideration and with no destabilizing loads, the non-dimensional beam slenderness $\bar{\lambda}_{LT}$ may be conservatively obtained from Table 2.3. Table 2.3 has been derived on the basis of Equation (2.21) with the conservative assumptions of $C_1 = 1.0$, $U = 0.9$, $V = 1.0$, $D = 1.0$ and $\sqrt{\beta_w} = 1.0$.

Table 2.3
$\bar{\lambda}_{LT}$ for different steel grades

S235	S275	S355
$\bar{\lambda}_{LT} = \frac{L / i_z}{104}$	$\bar{\lambda}_{LT} = \frac{L / i_z}{96}$	$\bar{\lambda}_{LT} = \frac{L / i_z}{85}$

Note that the simplified method described in this Section can lead to more favourable results if in-plane curvature prior to buckling is accounted for in the calculation of the parameter U (through the parameter g described in Section 2.2.1). The slenderness would be less than that derived from Equation (2.18) using a simplified value of M_{cr} that neglects this beneficial effect.

2.4 Equivalent uniform moment factors C_1

The distribution of bending moments along the length of a beam influences the value of the elastic critical moment. Allowance for the variation of bending moments on the elastic buckling moment M_{cr} or slenderness $\bar{\lambda}_{LT}$ of a beam may be made by means of the equivalent uniform moment factor C_1 (see Equations (2.14) and (2.21)). Uniform bending moment is the most severe scenario, for which $C_1 = 1$. Taking $C_1 = 1$ is also conservative for other patterns of moment, but may become overly conservative when the bending moment distribution varies significantly from uniform.

Recommended values of C_1 and $1/\sqrt{C_1}$ are given in Table 2.4 and Table 2.5. These values are taken from P362[12]. Similar values are also available elsewhere including

ψ	C_1	$1/\sqrt{C_1}$
1.00	1.00	1.00
0.75	1.17	0.92
0.50	1.36	0.86
0.25	1.56	0.80
0.00	1.77	0.75
-0.25	2.00	0.71
-0.50	2.24	0.67
-0.75	2.49	0.63
-1.00	2.76	0.60

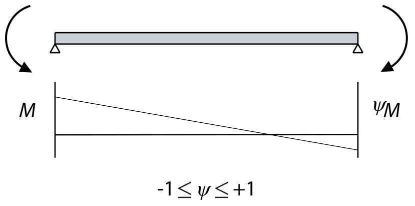


Table 2.4 Values of of C_1 and $1/\sqrt{C_1}$ for end moment loading

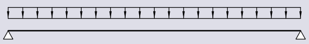
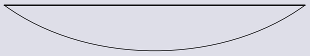
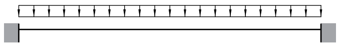
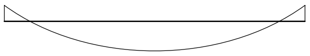
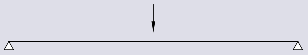
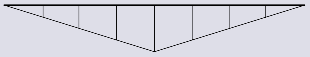
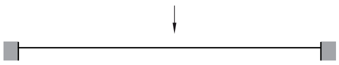
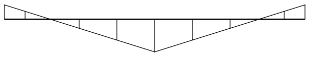
LOADING AND SUPPORT CONDITIONS	BENDING MOMENT DIAGRAM	C_1	$1/\sqrt{C_1}$
		1.13	0.94
		2.60	0.62
		1.35	0.86
		1.69	0.77

Table 2.5 Values of C_1 and $1/\sqrt{C_1}$ for cases with transverse loading

in NCCI SN002[7] but differences between sources will exist owing to the approximate (numerical) origins of the values. Guidance on effective length parameters for beams is provided in the NCCI SN009[13]; further discussion is contained in Section 3.1.

2.5 Destabilizing loads

When a beam is loaded vertically such that the load can move with the beam as it buckles, the level of application of the load relative to the shear centre of the beam becomes important. Note that for doubly-symmetric sections, the shear centre and the centroid coincide. Loads applied above the shear centre of the beam have a 'destabilizing' effect, resulting in lower values of M_{cr}, while loads applied below the shear centre have a 'stabilizing' effect, resulting in higher values of M_{cr}.

Introducing the non-dimensional load level parameter $\eta = 2z_g/h_f$, in which h_f is the distance between the centres of the flanges of the beam, three cases may be considered for a simply-supported beam with a uniformly distributed load:

1. load applied at the shear centre ($\eta = 0$),
2. load applied at the level of the top flange ($\eta = 1$), and
3. load applied at the level of the bottom flange ($\eta = -1$).

The elastic buckling moments M_{cr} for cases (2) and (3) may be normalized by that for case (1), denoted $M_{\mathrm{cr},\eta=0}$, to illustrate the effect of load level, which is then expressed as:

$$\frac{M_{\mathrm{cr}}}{M_{\mathrm{cr},\eta=0}} = \frac{\sqrt{1+\frac{1}{\kappa}+\left(C_2\eta\right)^2}-C_2\eta}{\sqrt{1+\frac{1}{\kappa}}} \tag{2.26}$$

where

κ $= (\pi^2 EI_{\mathrm{w}} / GI_{\mathrm{T}}L^2)$ (this is π^2 times the parameter k in SN003).

For a value of $\kappa = 0$, SN003 gives the values of C_2 as:

C_2 $= 0.454$ for a simply-supported beam with a uniformly distributed load

C_2 $= 0.630$ for a simply-supported beam with a central point load

Curves showing the variation of $M_{\mathrm{cr}}/M_{\mathrm{cr},\eta=0}$ are shown in Figure 2.4 for a range of values of κ. Values of κ will typically range between about 0.4 and 2.

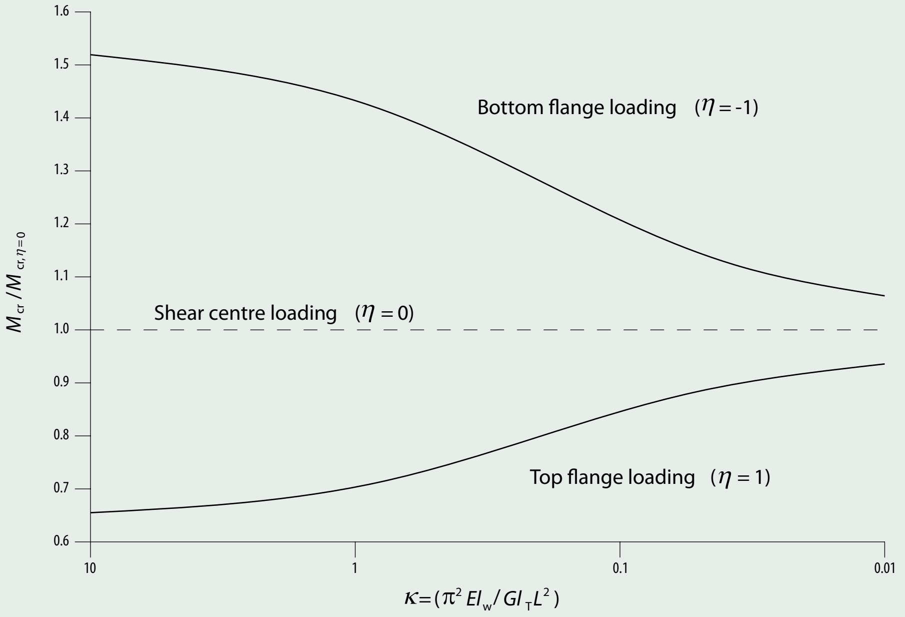


Figure 2.4
Effect of load level on elastic buckling moment M_{cr}

Figure 2.4 shows that top flange loading gives lower values for M_{cr} for all geometries, though the effect of load level becomes less pronounced as the member length increases. The destabilizing effects may be explained with reference to Figure 2.5. As the twisting associated with buckling starts, so the line of action of the load becomes eccentric to the beam's shear centre. This induces a destabilizing effect in the form of a moment equal to $F \times e$ where e is the eccentricity of the load F. As buckling develops this destabilizing effect increases (since e increases), thereby tending to accelerate the process.

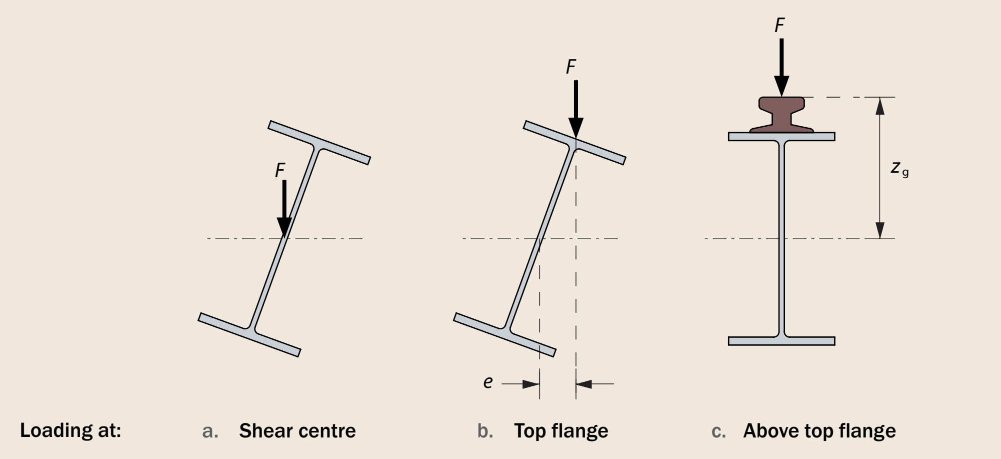


Figure 2.5
Effect of load level

The destabilizing effect may be assessed accurately by including the level of load application above the shear centre z_g in the calculation of M_{cr} (Equation (2.14)), or by inclusion of the full destabilizing parameter D (Equation (2.25)) in the calculation of $\bar{\lambda}_{LT}$ (Equation (2.21)) and hence $M_{b,Rd}$. Values for the destabilizing parameter D for a range of support and loading scenarios may be found in the NCCI SN009[13]. For example, for the basic case of a beam whose ends are prevented from twisting and deflecting laterally with no additional end restraint (so both flanges are free to rotate on plan), $D = 1.2$ to reflect the influence of top flange loading. This approximation generally gives a higher value of $\bar{\lambda}_{LT}$ (i.e. is more conservative) than that derived from Equation (2.25) or (2.14).

It is, of course, fundamental to the discussion in this Section that the loading is applied in such a way that as the beam tends to buckle sideways the loading can move freely with it. If the load cannot move in this way the eccentricity e shown in Figure 2.5(b) will not develop and no destabilizing effect will be present. Thus in practice, destabilizing loads are only considered in cases for which the applied loading offers no resistance to lateral movement, e.g. a free standing brick wall on a beam. Normal loads from floors do not constitute a destabilizing load.

2.6 Effect of intermediate lateral restraint

2.6.1 Restraint of columns

The behaviour of both columns and beams when provided with some form of intermediate lateral restraint may best be illustrated by taking the example of an axially loaded column and considering two situations of a central restraint and a distributed restraint, as illustrated in Figure 2.6. The central 'spring' restraint may or may not constrain the deformation of the column at mid-height. Considering the equilibrium of a centrally braced column of length L leads to the approximate relationship between the enhanced elastic critical force N_{cr} and the stiffness of the spring restraint K_T as:

$$N_{cr} = N_E + \frac{3}{16} K_T L \qquad (2.27)$$

where

$N_E = \dfrac{\pi^2 EI}{L^2}$, the Euler load of the unrestrained column.

If K_T exceeds a value of:

$$K_T = \frac{16\pi^2 EI}{L^3} = \frac{16N_E}{L} \tag{2.28}$$

the column will buckle in the second mode (two half sine waves), with the result that further increases in K_T will not produce corresponding increases in N_{cr}[14,15]. The column therefore effectively becomes restrained at its mid-height, and $N_{cr} = 4N_E$ (see Figure 2.6).

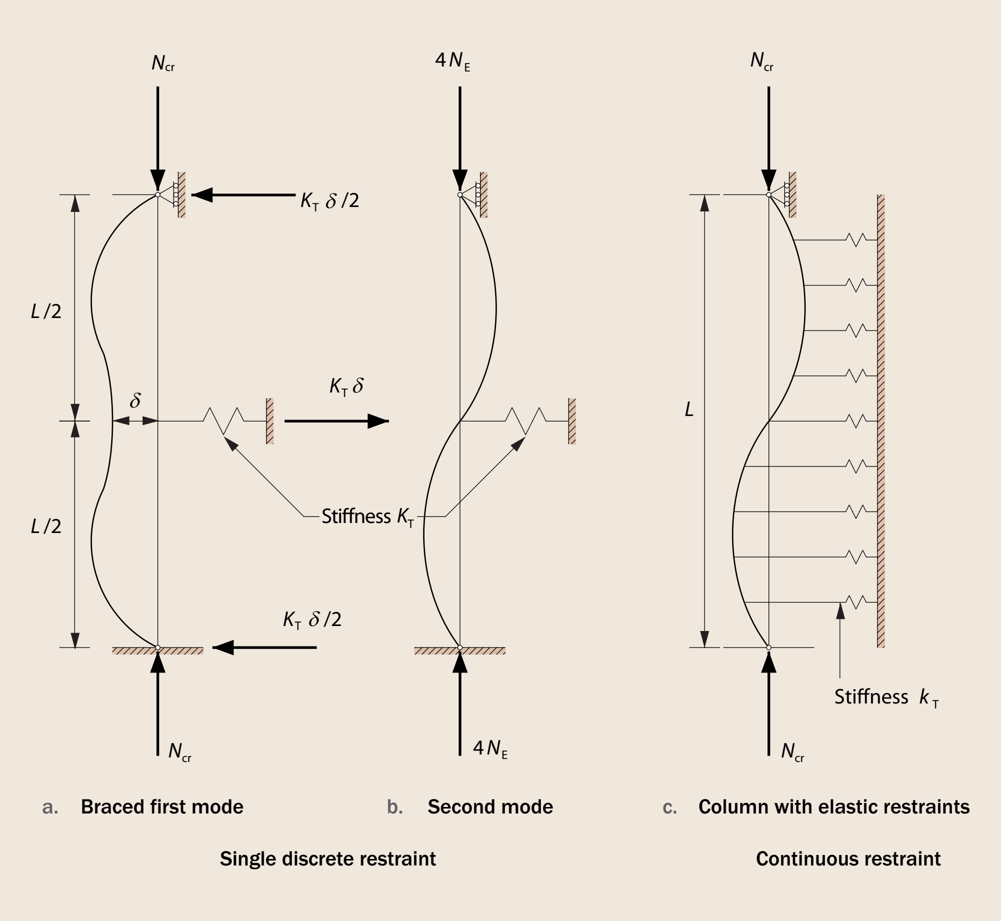

N_{cr}
$4N_E$
N_{cr}
$K_T\,\delta/2$
$L/2$
δ
$K_T\,\delta$
L
Stiffness K_T
$L/2$
$K_T\,\delta/2$
Stiffness k_T
N_{cr}
$4N_E$
N_{cr}
a. **Braced first mode**
b. **Second mode**
c. **Column with elastic restraints**
Single discrete restraint
Continuous restraint

Figure 2.6 Column with intermediate elastic restraint

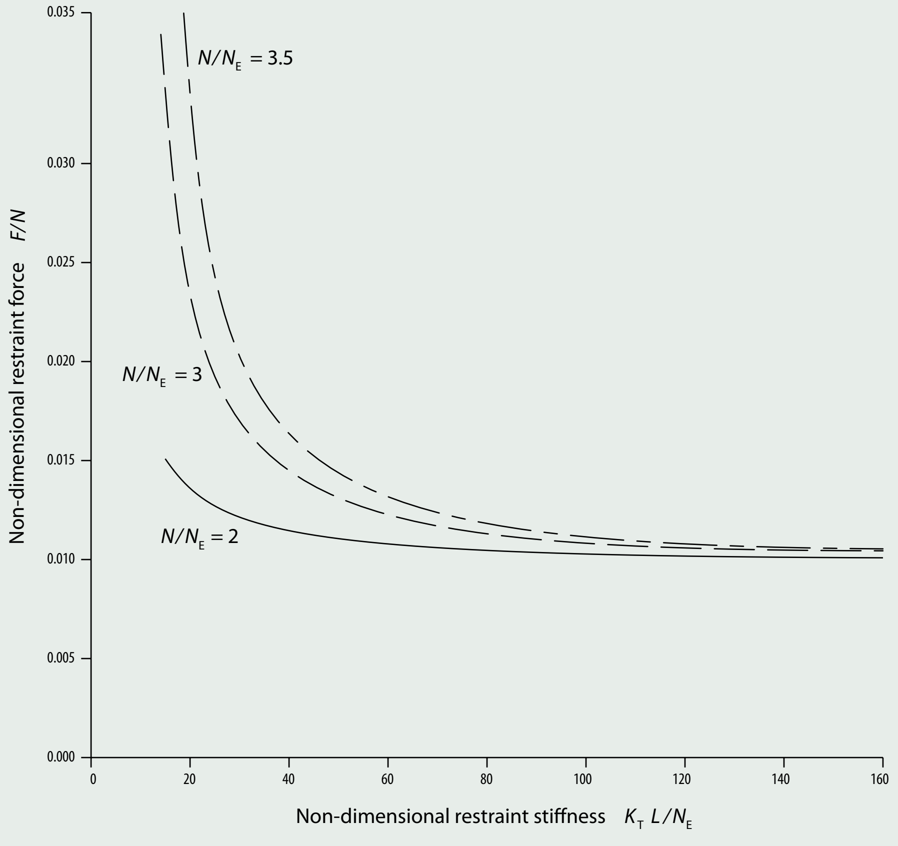
When a perfect column buckles in the second mode there is no force in the restraint. However, since columns are not perfectly straight, the spring stiffness K_T should be considerably higher than the value given by Equation (2.28) in order that the springs are fully effective as restraints and the forces that they attract are not excessive. A number of analytical studies of imperfect columns with intermediate elastic restraints have been performed,[16,17,18] and relationships between the forces experienced by the restraints and the load in the column have been derived.

For a pin-ended column with a single central intermediate lateral restraint and a sinusoidal imperfection of amplitude e_0, the ratio of the restraint force F to the axial load N in the member is given by[17]:

$$\frac{F}{N} = \frac{e_0 / L}{\left(\frac{N_\mathrm{E}}{N} - 1\right)} \times \frac{1}{\frac{N}{K_\mathrm{T} L} + \frac{1}{2\pi\sqrt{N / N_\mathrm{E}}} \tan\left(\frac{\pi}{2}\sqrt{\frac{N}{N_\mathrm{E}}}\right) - \frac{1}{4}} \qquad (2.29)$$

Similar expressions for other scenarios, including non-central restraints, multiple restraints and different imperfections and end conditions are also available[19].

Figure 2.7 shows the relationship between restraint force F and stiffness K_T for various levels of axial load N on a centrally restrained column. The graph has been derived from Equation (2.29), assuming the basic initial bow imperfection $e_0 = L/500$ given in Section 5.3.3 of BS EN 1993-1-1 when a single member is to be restrained; the typical fabrication tolerance for hot-rolled steelwork is $L/1000$. From Figure 2.7, it may be observed that a restraint stiffness of the minimum theoretical value required for a perfect column to buckle into two half sine waves ($K_\mathrm{T} = 16N_\mathrm{E}/L$) results in large restraint forces in the presence of an imperfection, and that to avoid such large restraint forces, the restraint stiffness has to be significantly greater than this value. This analysis provides a useful indication of the degree of bracing stiffness necessary in practice to restrain members against buckling.



Note: These curves have been derived assuming that $e_0 = L/500$

Figure 2.7 Relationship between restraint force and stiffness for various levels of axial load in a centrally restrained column

From Figure 2.7 it may be seen that restraint forces begin to grow rapidly for non-dimensional restraint stiffnesses $K_T L/N_E$ of less than about 60, while a value of approximately 120 is required to limit the restraint forces to just beyond 1% of the axial load in the column. Noting that the bending stiffness of a column (acting as a beam) is $48EI/L^3$, the non-dimensional restraint stiffness of 120 corresponds to a restraint stiffness of about 25 times this value, or 7.5 times the theoretical stiffness required to force a perfect, centrally restrained column into the second mode.

For the case of continuous elastic restraint shown in Figure 2.6(c), the relationship between N_{cr} and k_T is defined [14] as follows:

$$N_{cr} = N_E \sqrt{k_T L^2 / \pi^2 N_E} \left[\frac{m^2}{\sqrt{k_T L^2 / \pi^2 N_E}} + \frac{\sqrt{k_T L^2 / \pi^2 N_E}}{m^2} \right] \tag{2.30}$$

where k_T is the stiffness of restraint per unit length of the column and m is the number of half-waves into which the column buckles.

In this case, N_{cr} will continue to increase with increasing values of k_T as shown in Figure 2.8. Moreover, progressive changes in buckled shape will occur, with the column buckling into an increasing number of half-waves as the stiffness of the restraint is increased. Thus no equivalent of a limiting value of K_T as defined by Equation (2.28) exists.

However, a lower bound result for all cases, as shown in Figure 2.8, is obtained from:

$$N_{cr} = 2\sqrt{k_T EI} \tag{2.31}$$

For a series of discrete restraints at a spacing L_R, the stiffness $k_T L_R$ required to achieve $N_{cr} = \pi^2 EI/L_R^2$ is given by rearrangement of Equation (2.31) as:

$$k_T L_R = \pi^4 EI/4L_R^3 = 24.3\ EI/L_R^3$$

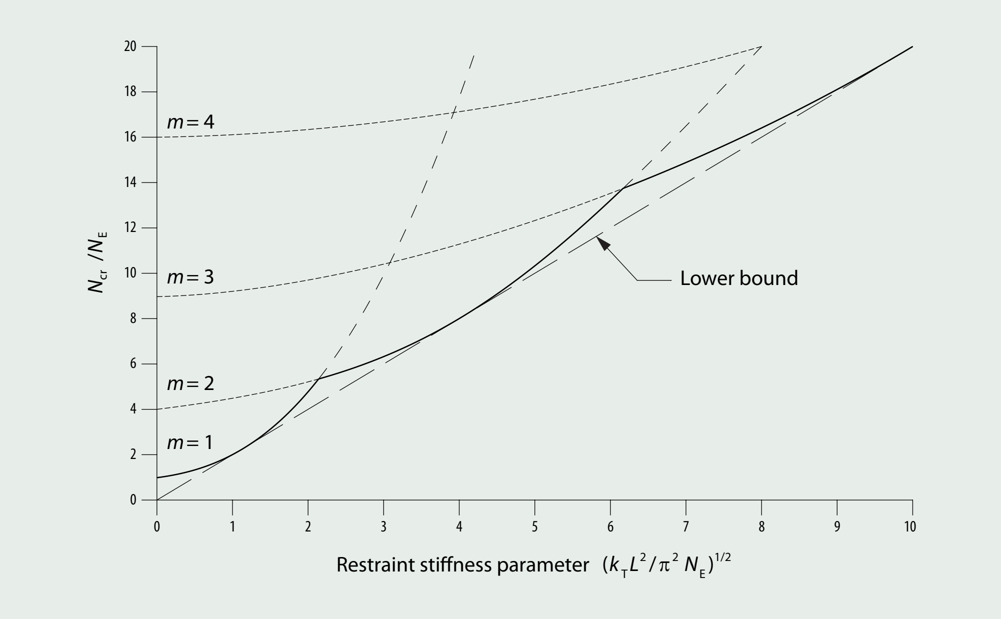


Figure 2.8 Effect of bracing stiffness on critical load for the arrangement shown in Figure 2.6

It may be noted that a minimum stiffness for intermediate restraints of $40\ EI/L_R^{\ 3}$ is given in PD 6695-2[20], Clause 5.3.

2.6.2 Restraint of beams

Similar analyses may be performed for beams[21] to obtain analogous relationships between M_{cr} and restraint stiffness. In this case, the problem is inherently more complex because restraints may act to limit lateral deflection, twist, or both, and beams may be subject to a variety of different forms of loading.

However, the different buckling characteristics of beams in comparison to columns leads to an appropriate minimum restraint stiffness of around 10 times the lateral bending stiffness of the beam when bracings are attached to the compression flange[22].

2.7 Compression or tension flange restraints

Cases of practical significance for beams are often likely to involve the restraint being provided to one of the flanges through attachment to a floor or roof. Often this restraint may be assumed to be continuous even if the actual arrangement is one of a series of interconnections at close intervals. For a simply supported beam under gravity loading the restraining effect will be a maximum when the top (compression) flange is laterally restrained, or sufficient torsional restraint is provided that the top flange cannot displace laterally. In many cases, the beam may be designed as being fully restrained, achieving its full in-plane bending resistance.

Frequently, restraint is provided only to the tension flange. An example is a beam supporting roof sheeting which is under suction loading. While the restraining effect is likely to be significantly reduced, benefits, in terms of improved member stability, may still be derived. If the tension flange restraint has high lateral stiffness, but insignificant torsional stiffness, buckling of the type illustrated in Figure 2.9 may occur, with the beam simply twisting about the restrained longitudinal axis[23]. Clearly no change in behaviour will result from any further increase in lateral stiffness of the bracing. Some benefit could, however, be achieved if the bracing were also able to supply a measure of torsional restraint, thereby further restricting the torsional deformations of the beam.

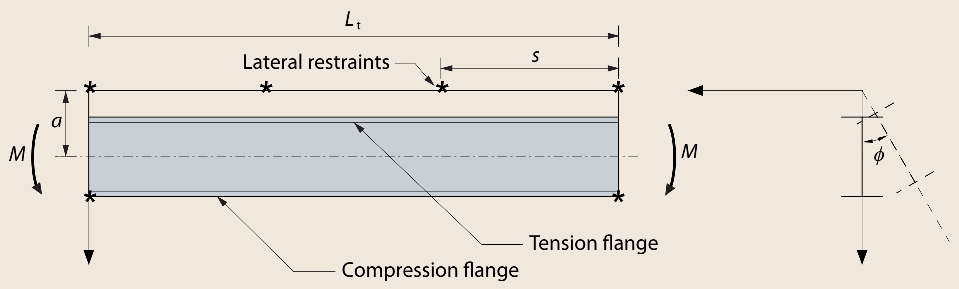


Figure 2.9 Buckling about a restrained axis adjacent to the tension flange

One case of particular practical significance is that of a portal frame rafter supported laterally at relatively close intervals by purlins whose axial stiffness is sufficient to prevent lateral deflection at each braced point. Neglecting axial force in the rafter and assuming that the bending stiffness of the purlin plus its connection offers rotational restraint to the rafter, then the rafter may buckle in one of two modes[23]:

1. an overall buckling mode involving twisting about the braced axis at a value of $M_{\mathrm{cr,T}}$ given by:

   $$M_{\mathrm{cr,T}} = \frac{1}{2a}\left[\frac{n_{\mathrm{b}}^2\pi^2 a^2 EI_{\mathrm{z}}}{L_{\mathrm{t}}^2} + \frac{n_{\mathrm{b}}^2\pi^2 EI_{\mathrm{w}}}{L_{\mathrm{t}}^2} + GI_{\mathrm{T}} + C_{\vartheta,\mathrm{k}}\frac{L_{\mathrm{t}}^2}{n_{\mathrm{b}}^2\pi^2}\right] \tag{2.32}$$

   where

   n_{b} is the number of half-waves within the length L_{t} for which $M_{\mathrm{cr,T}}$ is a minimum

   a is the distance between the restrained longitudinal axis (e.g. the centroid of the purlins) and the shear centre of the member

   $C_{\vartheta,\mathrm{k}}$ is the rotational stiffness (per unit length of beam) provided by the restraint (e.g. purlins and roof sheeting). Guidance on the determination of $C_{\vartheta,\mathrm{k}}$ is given in Clause BB.2.2 of BS EN 1993-1-1.

   L_{t} is the length of member between effective torsional restraints.

2. between braced points in a lateral torsional buckling mode at a value of M_{cr} given by Equation (2.33), which is the same as Equation (2.12) with L replaced by s, where s is the spacing between the intermediate tension flange restraints.

   $$M_{\mathrm{cr}} = \frac{\pi}{\mathrm{s}}\sqrt{EI_{\mathrm{z}}GI_{\mathrm{T}}}\sqrt{1+\frac{\pi^2 EI_{\mathrm{w}}}{s^2 GI_{\mathrm{T}}}} \tag{2.33}$$

Conservatively assuming the torsional stiffness of the intermediate tension flange restraints to be zero and taking n_{b} as unity, the elastic buckling moment of a uniform I section member with equal flanges under uniform bending is given by:

$$M_{\mathrm{cr,T}} = \frac{1}{2a}\left[\frac{\pi^2 a^2 EI_{\mathrm{z}}}{L_{\mathrm{t}}^2} + \frac{\pi^2 EI_{\mathrm{w}}}{L_{\mathrm{t}}^2} + GI_{\mathrm{T}}\right] \tag{2.34}$$

This equation is applicable to beams or lengths of beam between torsional restraints (i.e. restraints to both flanges), with intermediate lateral restraints to the tension flange that are sufficiently closely spaced so that the stability of the beam is maintained between these intermediate restraints. The latter can be ensured by adjusting the restraint spacing such that the elastic buckling moment from Equation (2.33), controlled by the spacing s of the intermediate tension flange restraints, is greater than that from Equation (2.34), controlled by the distance L_{t} between restraints to both flanges.

Under a linear moment gradient, $M_{\mathrm{cr,T}}$ may be determined from:

$$M_{\mathrm{cr,T}} = C_{\mathrm{m}}\frac{1}{c^2}\frac{1}{2a}\left[\frac{\pi^2 a^2 EI_{\mathrm{z}}}{L_{\mathrm{t}}^2} + \frac{\pi^2 EI_{\mathrm{w}}}{L_{\mathrm{t}}^2} + GI_{\mathrm{T}}\right] \tag{2.35}$$

where C_m accounts for linear moment gradients and may be determined from Clause BB.3.3.1 of BS EN 1993-1-1 and c accounts for any taper of the beam and may be determined from Clause BB.3.3.3 of BS EN 1993-1-1.

For a nonlinear moment gradient, $M_\mathrm{cr,T}$ is given by:

$$M_\mathrm{cr,T} = C_\mathrm{n}\frac{1}{c^2}\frac{1}{2a}\left[\frac{\pi^2 a^2 EI_\mathrm{z}}{L_\mathrm{t}^2} + \frac{\pi^2 EI_\mathrm{w}}{L_\mathrm{t}^2} + GI_\mathrm{T}\right] \tag{2.36}$$

where C_n accounts for nonlinear moment gradients and may be obtained from Clause BB.3.3.2 of BS EN 1993-1-1.

The above may be recast in terms of a stable length L_k of beam with intermediate tension flange restraint between points of torsional restraint (i.e. restraint to both flanges). For uniform moment, lateral torsional buckling effects may be ignored provided the length of beam between adjacent torsional restraints does not exceed the maximum stable length L_k given by Equation (2.37), which is Equation (BB.6) of BS EN 1993-1-1.

$$L_\mathrm{k} = \frac{\left(5.4 + \dfrac{600 f_\mathrm{y}}{E}\right)\left(\dfrac{h}{t_\mathrm{f}}\right) i_\mathrm{z}}{\sqrt{5.4\left(\dfrac{f_\mathrm{y}}{E}\right)\left(\dfrac{h}{t_\mathrm{f}}\right)^2 - 1}} \tag{2.37}$$

Under a linear moment gradient and axial compression, the length of the member between torsional restraints should not exceed the maximum stable length L_s, given by Equation (2.38), which is Equation (BB.7) of BS EN 1993-1-1.

$$L_\mathrm{s} = \sqrt{C_\mathrm{m}}\, L_\mathrm{k}\left(\frac{M_\mathrm{pl,y,Rk}}{M_\mathrm{N,y,Rk} + aN_\mathrm{Ed}}\right) \tag{2.38}$$

where C_m accounts for linear moment gradients and may be obtained from Clause BB.3.3.2 of BS EN 1993-1-1, $M_\mathrm{pl,y,Rk}$ is the characteristic plastic moment resistance of the cross-section about the y-y axis (which is equal to the design plastic moment resistance $M_\mathrm{pl,y,Rd}$ when γ_M0 = 1.0, as is the case in both BS EN 1993-1-1 and the UK National Annex) and $M_\mathrm{N,y,Rk}$ is the characteristic plastic moment resistance of the cross-section about the y-y axis with reduction due to the axial load N_Ed.

For a nonlinear moment gradient, lateral torsional buckling may be ignored, provided the length of the member between torsional restraints does not exceed the maximum stable length L_s, given by Equation (2.39), which is Equation (BB.8) in BS EN 1993-1-1.

$$L_\mathrm{s} = \sqrt{C_\mathrm{n}}\, L_\mathrm{k} \tag{2.39}$$

Stability of the member between the intermediate tension flange restraints themselves may also be verified by considering a stable member length. Lateral torsional buckling between intermediate tension flange restraints may be ignored provided the spacing s of the restraints does not exceed the maximum stable length L_m, given by Equation (2.40), which is Equation (BB.5) of BS EN 1993-1-1. Units of N and mm must be used in this equation.

$$L_{\mathrm{m}} = \frac{38 i_z}{\sqrt{\frac{1}{57.4}\left(\frac{N_{\mathrm{Ed}}}{A}\right) + \frac{1}{756 C_1^2}\left(\frac{W_{\mathrm{pl,y}}^2}{A I_{\mathrm{T}}}\right)\left(\frac{f_{\mathrm{y}}}{235}\right)^2}} \tag{2.40}$$

where C_1 may be taken from Table 2.4 or Table 2.5.

2.8 Requirements for restraint stiffness and strength

For bracing against buckling, two requirements may be identified for all restraint systems:

1. Sufficient stiffness to increase the buckling load of the restrained member to the desired level by limiting the buckling deformations.
2. Sufficient strength to resist the loads transmitted as a result of restricting the buckling deformations.

The interrelationship between restraint stiffness and strength was highlighted in Figure 2.7, where it may be seen that the greater the stiffness of the restraint, the smaller its required strength [24]. Despite the importance of both strength and stiffness, many structural design codes provide only strength requirements (e.g. BS 5950-1) and it is assumed that a member of such strength will also possess sufficient stiffness.

BS EN 1993-1-1 defines an initial geometric imperfection for restrained members (Clause 5.3.3(1) of BS EN 1993-1-1) and gives a local force at splices (Clause 5.3.3(4) of BS EN 1993-1-1). The initial geometric imperfection may be replaced by an equivalent stabilizing force q_d, defined by Equation 5.13 of BS EN 1993-1-1, which is applied as a uniformly distributed load to be resisted by the bracing system. The local force at splices and the restraint force q_d arising from the initial geometric imperfection need not be applied together but both should be considered in conjunction with any external loads (e.g. wind load) on the bracing system (see Clause 5.3.3(5) of BS EN 1993-1-1). Braces at plastic hinge locations have more onerous requirements, as discussed in Section 5.

The equivalent stabilizing force q_d is defined in Clause 5.3.3(2) of BS EN 1993-1-1 as:

$$q_{\mathrm{d}} = \sum N_{\mathrm{Ed}} \, 8 \frac{e_0 + \delta_{\mathrm{q}}}{L^2} \tag{2.41}$$

in which N_{Ed} is the axial force in the compression flange of the beam, taken as $N_{Ed} = M_{Ed} / h$, where M_{Ed} is the maximum moment in the beam and h is the overall beam depth, e_0 is the initial imperfection in the beam, δ_q is the deflection of the bracing system under q_d plus any external loads and L is the length of the beam. For restraint to multiple beams, a summation of the equivalent stability forces is made for all beams being supported by the bracing system under consideration.

The member imperfection e_0 is defined by Equation 5.12 of BS EN 1993-1-1 as:

$$e_0 = \alpha_{\mathrm{m}} L / 500 \qquad (2.42)$$

where α_{m} is a reduction factor when multiple beams are being restrained, given in Clause 5.3.3(1) of BS EN 1993-1-1 as:

$$\alpha_{\mathrm{m}} = \sqrt{0.5\left(1 + \frac{1}{m}\right)} \qquad (2.43)$$

in which m is the number of members to be restrained.

Three analysis options are available for determining restraint forces due to initial geometric imperfections in the restrained members:

- First order analysis of bracing system under equivalent stabilizing force q_{d} ($\delta_{\mathrm{q}} \neq 0$) and any external loads
- Second order analysis of bracing system with equivalent stabilizing force q_{d} ($\delta_{\mathrm{q}} = 0$) and any external loads
- Second order analysis of bracing system with initial geometric imperfection e_0 and any external loads.

Of the three options, the first will typically be favoured since it does not require initial geometric imperfections to be incorporated into the structural model and it does not require second order analysis. This is the approach employed in Worked Example 1. The approach does, however, require iteration since the equivalent stabilizing force depends on the deflection of the bracing system. Two steps will generally be sufficient. In the first step, the in-plane deflection of the bracing system may be found under q_{d} (with $\delta_{\mathrm{q}} = 0$) and any external loads. In the second step, having now determined δ_{q}, a revised equivalent stabilizing force may be calculated. A flowchart to describe this process is given in SF024 [25].

Considering the basic imperfection of $L/500$ and assuming (for illustration purposes) the bracing system to be infinitely stiff, such that δ_{q} is zero, leads to a total force to be restrained ($q_{\mathrm{d}}L$) of 1.6% of N_{Ed} (from Equation (2.41)). For a beam laterally restrained at its supports with a single central restraint, this results in a central restraint force of 1% of N_{Ed} (with the remaining force being transferred equally to the end supports). It must be stressed, however, that these restraint forces assume that the bracing system is infinitely stiff. Since this cannot be the case, actual restraint forces will be greater than these values and will depend on the stiffness of the bracing system. The influence of the stiffness of the bracing system has been illustrated in Figure 2.7, where it may be seen that, in the presence of an imperfection of $L/500$, the force in a central restraint tends to approximately 1% of N_{Ed} for high bracing stiffness (i.e. the same value that emerges from Equation (2.41) when $\delta_{\mathrm{q}} = 0$). However, at lower levels of bracing stiffness, higher forces arise in the restraints. It is therefore important that account is taken of the deflection of the bracing system itself. Further, it must be ensured that all restraint forces are transferred to some ‘stiff’ point in the structure, for example, to in-plane bracing or concrete core walls.

For design purposes, a useful approach is to assume initially (and subsequently confirm) that the deflection of the bracing system δ_q will be less than a fairly conservative (i.e. large) value of $L/2000$; the total resulting equivalent stabilising force ($q_d L$) is then 2% of N_{Ed}. Provided that the actual deflection of the bracing system under this restraint force plus any other external loading (e.g. wind) is less than $L/2000$, a safe result will have been achieved and no iteration is required.

PRACTICAL EXAMPLES OF END & INTERMEDIATE RESTRAINTS TO BEAMS

This section describes a range of practical examples of beams with different end and intermediate restraint conditions, and illustrates how the rules set out in Eurocode 3, and introduced in Section 2 of this publication, are applied.

3.1 End restraint to beams

Several types of end conditions to beams that are found in practice require a degree of interpretation when deciding on the effective length L_{cr} $(= kL)$ to be used in calculating $\bar{\lambda}_{LT}$ or M_{cr}. One aspect is the (usually) non-quantifiable influence of the rotational flexibility on plan of the connection between the beam and the columns in framed construction. Another important consideration is the relative size/stiffness of the members. The following guidance applies for members typical of medium span construction (i.e. it does not apply to heavy beams connected to small columns).

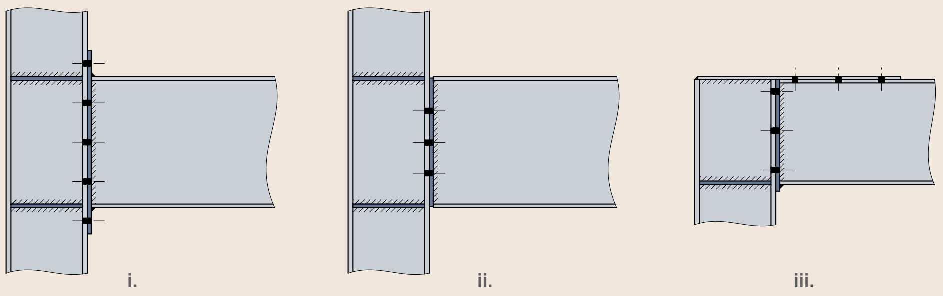
Several examples of practical end conditions are shown in Figure 3.1; they have been grouped as follows:

- Case (1) where the beam flanges may be assumed to be fully restrained on plan (Figure 3.1(a))
- Case (2) where the flanges are partially restrained on plan (Figure 3.1(b))
- Case (3) where the flanges are free to rotate on plan (Figure 3.1(c)).

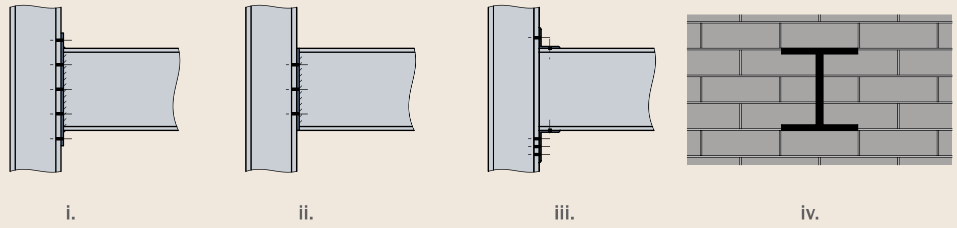
Two further cases, not illustrated, are Cases (4) and (5) which are similar to Cases (1) and (2), respectively, but with restraint applied only to the compression flange at the beam ends.

Recommended effective length parameters are given in Table 3.1.

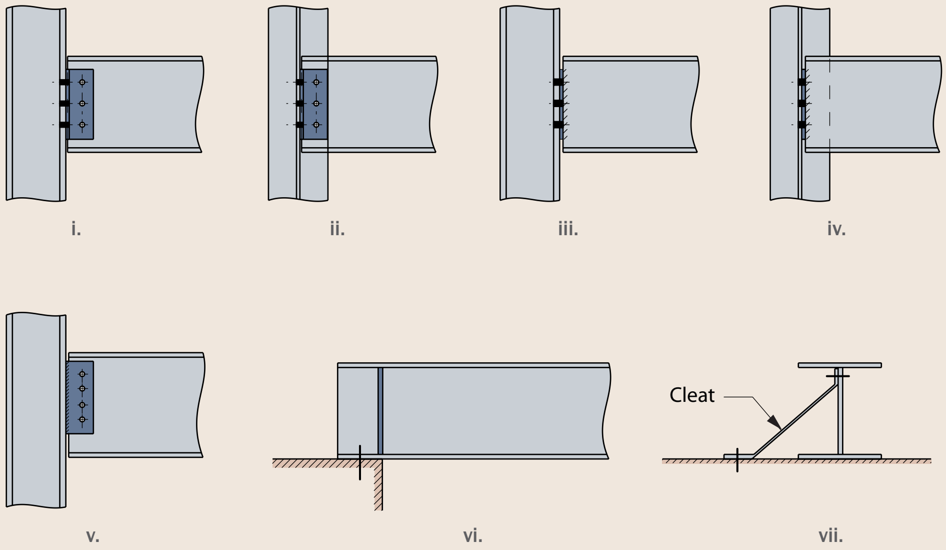
- End plate or moment resisting connections with column web stiffeners offer the most rotational restraint in the minor axis direction and are covered by Case (1). Where stiffeners are not provided, Case (2) is more appropriate.
- Web cleats or similar connections offer little rotational restraint and are covered by Case (3).
- Beams built into walls may be considered to be partially restrained.
- Beam-to-beam connections are usually of the form shown in Case (3). Relatively little rotational restraint is provided at the beam ends.



a. **Case (1): Flanges fully restrained on plan**



b. **Case (2): Flanges partially restrained on plan**



c. **Case (3): Flanges free to rotate on plan**

Figure 3.1
Common cases of restraint at beam ends

- Notches in the beam may affect the restraint offered to the flange of the beam, causing an increase in its effective length. This is treated as a special case and is covered in Section 3.4.
- For the destabilizing load condition, the destabilizing parameter D should be taken as 1.2, otherwise D should be taken as 1.0. Alternatively, if calculating $\overline{\lambda}_{\mathrm{LT}}$ via M_{cr}, the distance between the applied load and the shear centre z_{g} may be used in Equation (2.14). See also Section 4 of this guide.
- Where restraint conditions at the beam ends differ, the mean value of k should be used.

Table 3.1 Effective length parameters k

CONDITIONS OF RESTRAINT AT SUPPORTS		RESTRAINT CASES	k
Compression flange laterally restrained. Nominal torsional restraint against rotation about longitudinal axis.	Both flanges fully restrained against rotation on plan	Case (1): Figure 3.1(a)	0.7
	Both flanges partially restrained against rotation on plan	Case (2): Figure 3.1(b)	0.8
	Both flanges free to rotate on plan	Case (3): Figure 3.1(c)	1.0
	Compression flange fully restrained against rotation on plan	Case (4): Figure 3.1(a)*	0.75
	Compression flange partially restrained against rotation on plan	Case (5): Figure 3.1(b)*	0.85

The above values have been extracted from Table 4.1 of the NCCI SN009[13]

* With compression flange restraint only

3.2 Cantilever beams

The stability of a cantilever is significantly affected by the restraint conditions at both the support and tip. The influence of destabilizing loading is also pronounced. Guidance for common support conditions is given in NCCIs SN006[26] and SN009[13]; both are discussed below.

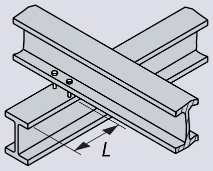
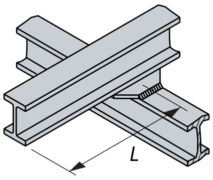
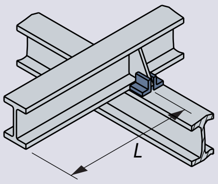
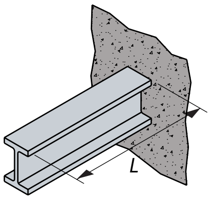
The lateral torsional buckling resistance of a cantilever is determined on the basis of the member slenderness $\bar{\lambda}_{\mathrm{LT}}$, which may be calculated either by means of M_{cr} (Equation (2.14)) or more directly through Equation (2.21).

Formulae for the determination of M_{cr} for cantilever beams are given in NCCI SN006. Lateral and torsional restraints are assumed at the support and two conditions of warping restraint are considered – free or restrained. Both tip loading and distributed loading acting either individually or in combination are covered; the level of load application, relative to the shear centre of the beam, is also considered.

Effective length factors k and destabilizing parameters D to be used in the calculation of member slenderness $\bar{\lambda}_{\mathrm{LT}}$ (Equation (2.21)) for cantilevers without intermediate restraints but with a variety of restraint conditions at the support are presented in NCCI SN009, and repeated in Figure 3.2. For normal loading conditions, $D = 1.0$.

For optimum performance, lateral restraint should be provided to both flanges of the beam at the support – see Cases (c) and (d) in Figure 3.2. Other cantilever configurations that provide effective moment continuity and lateral and torsional restraint at the support, which may be treated similarly to Case (c) in Figure 3.2, are shown in Figure 3.3.

For cantilevers with intermediate lateral restraints to the compression flange, the effective length parameter k may be taken as 1.0 and the length L should be taken as the distance between points of adjacent lateral restraint, provided that effective lateral

RESTRAINT CONDITIONS		k	D
AT SUPPORT	**AT TIP***		
a) Continuous, with lateral restraint to top flange 	1) Free	3.0	2.5
	2) Lateral restraint to top flange	2.7	2.8
	3) Torsional restraint	2.4	1.9
	4) Lateral and torsional restraint	2.1	1.7
b) Continuous, with partial torsional restraint 	1) Free	2.0	2.5
	2) Lateral restraint to top flange	1.8	2.8
	3) Torsional restraint	1.6	1.9
	4) Lateral and torsional restraint	1.4	1.7
c) Continuous, with lateral and torsional restraint 	1) Free	1.0	2.5
	2) Lateral restraint to top flange	0.9	2.8
	3) Torsional restraint	0.8	1.9
	4) Lateral and torsional restraint	0.7	1.7
d) Restrained laterally, torsionally & against rotation on plan 	1) Free	0.8	1.75
	2) Lateral restraint to top flange	0.7	2.0
	3) Torsional restraint	0.6	1.0
	4) Lateral and torsional restraint	0.5	1.0

*Tip restrain conditions

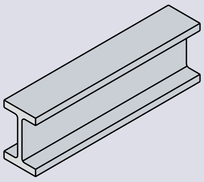
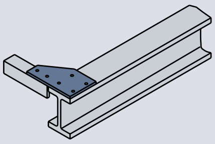
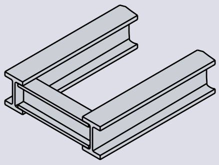
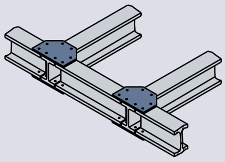
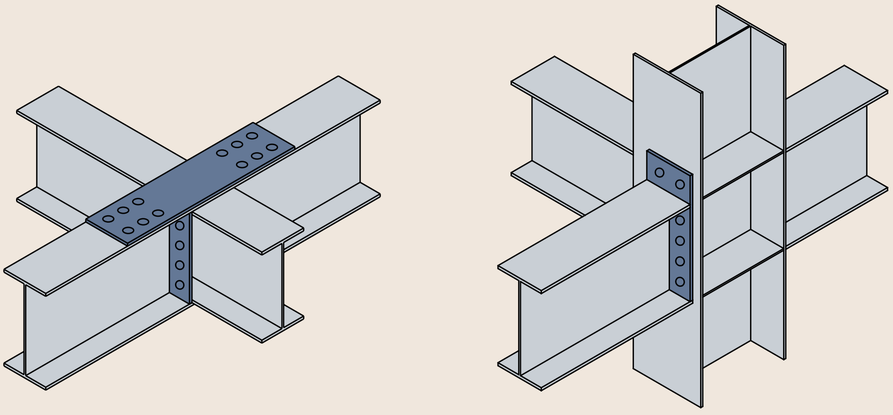
1) Free	2) Lateral restraint to top flange	3) Torsional restraint	4) Lateral and torsional restraint
			
(not braced on plan)	(braced on plan in at least one bay)	(not braced on plan)	(braced on plan in at least one bay)

Figure 3.2 Effective length parameter k and destabilizing parameter D for cantilevers without intermediate restraint



Discontinuous cantilever, at same level as continuous supporting beam

Cantilever supported from flange of column

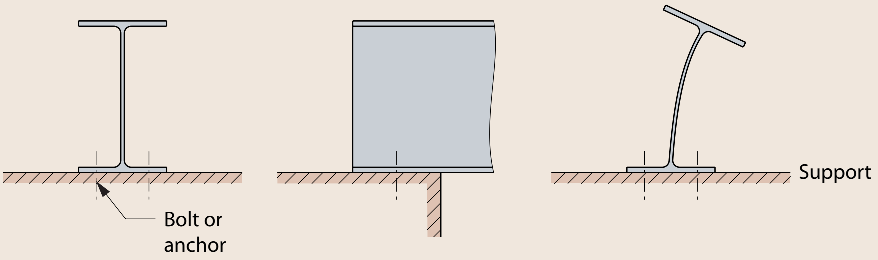
Figure 3.3
Other cantilever configurations that provide effective moment continuity and lateral and torsional restraint at the support

and torsional restraint is present at the support (examples of which are shown in Case (c) of Figure 3.2 and Figure 3.3) and the load is not destabilizing. (This guidance is taken from NCCI SN009[13].)

3.3 Beam supported at bottom flange only at the ends

Figure 3.4 illustrates the situation in which lateral restraint is only provided to the bottom flange of a beam at its end supports, while the top flange is unrestrained. This may result in distortion of the cross-section with web bending and consequently a reduction in member buckling resistance. Two cases commonly arise, the first where there is a positive connection between the bottom flange and the support (e.g. by bolting) and the second where no such connection exists and the beam simply bears on the support; the latter case results in a greater reduction in member resistance.

The situation where the only restraint at the support is at the bottom flange (bottom flange fixed to support) is not covered explicitly by BS EN 1993-1-1 and although there is currently no relevant NCCI the problem has been investigated (see *Buckling of beams supported on seats*[27]). The reduced buckling resistance may be accounted for by means of an increased effective length factor k. The following effective lengths kL may be used in the calculation of $\bar{\lambda}_{LT}$ or M_{cr} (Equation (2.13)) for simply-supported beams.



Side view

Buckled shape

Figure 3.4
Lateral restraint to bottom flange only at end support

In cases where there is positive connection to the bottom flange:

$kL = 1.0L + 2h$ for normal loading

$kL = 1.2L + 2h$ for top flange loading

where

L = length of beam between supports

h = depth of beam.

In cases where no positive connection between the bottom flange of the beam and the support is provided, i.e. restraint against torsion is due solely to bearing of the bottom flange on the supports, then:

$kL = 1.2L + 2h$ for normal loading

$kL = 1.4L + 2h$ for top flange loading

3.4 Beams notched at the ends

Notching or coping of the top flange or both flanges of a beam is often required to enable suitable end connections to be achieved, for example to other beams at right angles, as shown in Figure 3.5.

Two primary considerations arise as a result of the introduction of notches. The first relates to local buckling of the notched region of the beam, for which guidance, derived from References 28 and 29 and also adopted in Reference 30, is given below (see Section 3.4.1). The second relates to the overall lateral stability of unrestrained notched beams, where the presence of the notch leads to more flexible end conditions and therefore a reduction in the lateral torsional buckling resistance of the beam. Guidance for the latter case, derived from Reference 31 is presented in Section 3.4.2. Checks for both local buckling of the notched region and laterally stability of unrestrained notched beams are shown in Worked Example 2.

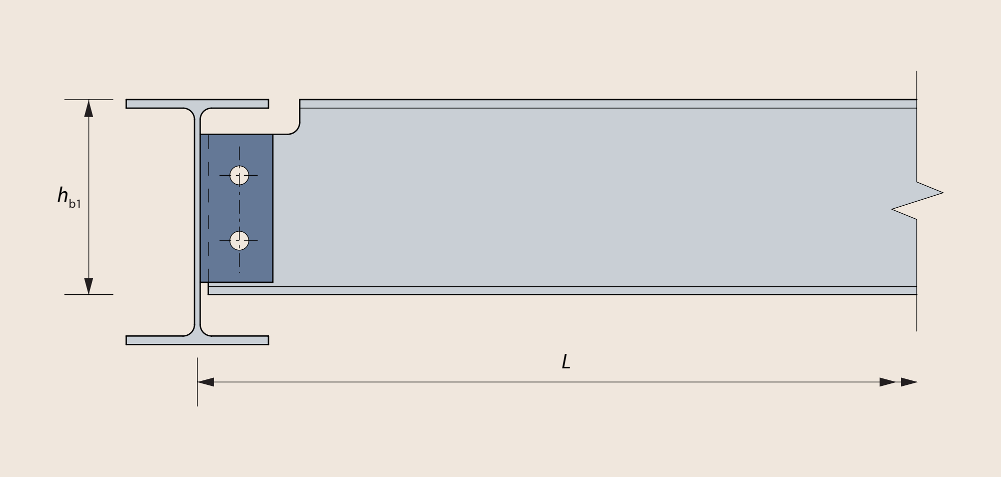


Figure 3.5 Notched beam (one flange notched)

3.4.1 Local buckling of notched region

Provided that the beam is restrained against lateral torsional buckling, no account need be taken of notch stability if the following conditions are met (see Figure 3.6 for definition of symbols):

For one flange notched:

$d_{\mathrm{nt}} \leq h_{\mathrm{b1}} / 2$ and,

$$l_{\mathrm{n}} \leq h_{\mathrm{b1}} \quad \text{for} \quad h_{\mathrm{b1}} / t_{\mathrm{w,b1}} \leq 55.1 \quad \text{(S275 steel)}$$

$$l_{\mathrm{n}} \leq \frac{167000 h_{\mathrm{b1}}}{(h_{\mathrm{b1}} / t_{\mathrm{w,b1}})^3} \quad \text{for} \quad h_{\mathrm{b1}} / t_{\mathrm{w,b1}} > 55.1 \quad \text{(S275 steel)}$$

$$l_{\mathrm{n}} \leq h_{\mathrm{b1}} \quad \text{for} \quad h_{\mathrm{b1}} / t_{\mathrm{w,b1}} \leq 48.5 \quad \text{(S355 steel)}$$

$$l_{\mathrm{n}} \leq \frac{114000 h_{\mathrm{b1}}}{(h_{\mathrm{b1}} / t_{\mathrm{w,b1}})^3} \quad \text{for} \quad h_{\mathrm{b1}} / t_{\mathrm{w,b1}} > 48.5 \quad \text{(S355 steel)}$$

For both flanges notched:

$\mathrm{Max}(d_{\mathrm{nt}}, d_{\mathrm{nb}}) \leq h_{\mathrm{b1}} / 5$ and,

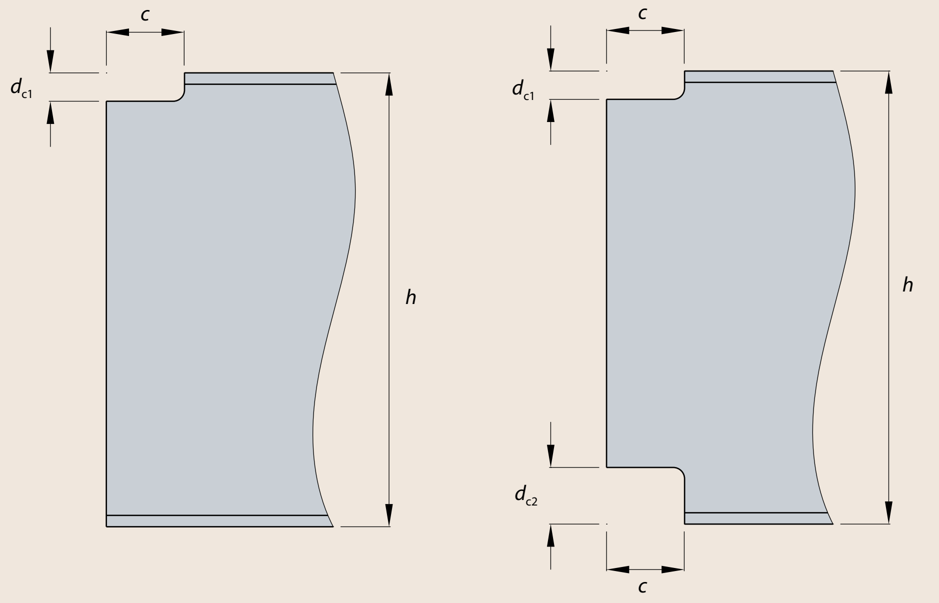
$$l_{\mathrm{n}} \leq h_{\mathrm{b1}} \quad \text{for} \quad h_{\mathrm{b1}} / t_{\mathrm{w,b1}} \leq 55.1 \quad \text{(S275 steel)}$$

$$l_{\mathrm{n}} \leq \frac{167000 h_{\mathrm{b1}}}{(h_{\mathrm{b1}} / t_{\mathrm{w,b1}})^3} \quad \text{for} \quad h_{\mathrm{b1}} / t_{\mathrm{w,b1}} > 55.1 \quad \text{(S275 steel)}$$

$$l_{\mathrm{n}} \leq h_{\mathrm{b1}} \quad \text{for} \quad h_{\mathrm{b1}} / t_{\mathrm{w,b1}} \leq 48.5 \quad \text{(S355 steel)}$$

$$l_{\mathrm{n}} \leq \frac{114000 h_{\mathrm{b1}}}{(h_{\mathrm{b1}} / t_{\mathrm{w,b1}})^3} \quad \text{for} \quad h_{\mathrm{b1}} / t_{\mathrm{w,b1}} > 48.5 \quad \text{(S355 steel)}$$

Where the notch length c exceeds these limits, either suitable stiffening should be provided or the notch should be verified using References 28, 29 and 32.



a. **One flange notched**

b. **Both flanges notched**

Figure 3.6 Details of notched region

3.4.2 Lateral stability of unrestrained notched beams

When a notched beam is laterally unrestrained, its overall lateral torsional buckling resistance should be verified. The following guidance is taken from Annex D of Reference 31 and applies to beams with one flange (the top flange) or both flanges notched. To reflect the reduced end restraint brought about by the introduction of notches, a reduced elastic buckling moment $M_{\mathrm{cr,n}}$ may be determined and subsequently used in the calculation of $\bar{\lambda}_{\mathrm{LT}}$ from Equation (2.18). The reduced elastic buckling moment $M_{\mathrm{cr,n}}$ may be calculated as:

$$M_{\mathrm{cr,n}} = r_{\mathrm{n}} M_{\mathrm{cr}} \tag{3.1}$$

in which M_{cr} is the elastic buckling moment of the unnotched beam (Equation (2.14)) and r_{n} is the reduction factor obtained from Equation (3.2) for UKB sections.

$$r_{\mathrm{n}} = 1.0 - \frac{\alpha}{C_1 L}\sqrt{\frac{EI_{\mathrm{w}}}{GI_{\mathrm{T}}}} \tag{3.2}$$

where

C_1 is the equivalent uniform moment factor from Table 2.4 or Table 2.5

L is the length of the notched beam – see Figure 3.5.

For normal loading:

With notching of the top flange:

$$\begin{aligned} \alpha &= \frac{h}{4(h-d_{\mathrm{c1}})} + \frac{2c}{h} \qquad \text{for } \frac{d_{\mathrm{c1}}}{h} \le 0.15 \\ \alpha &= \frac{h}{2(h-d_{\mathrm{c1}})} + \frac{1.5c}{h} \qquad \text{for } \frac{d_{\mathrm{c1}}}{h} > 0.15 \end{aligned} \tag{3.3}$$

With notching of both flanges:

$$\alpha = \frac{h}{3(h-[d_{\mathrm{c1}}+d_{\mathrm{c2}}])} + \frac{3.5c}{h} \tag{3.4}$$

For top flange loading:

With notching of the top flange:

$$\begin{aligned} \alpha &= \frac{h}{2(h-d_{\mathrm{c1}})} + \frac{2c}{h} \qquad \text{for } \frac{d_{\mathrm{c1}}}{h} \le 0.15 \\ \alpha &= \frac{h}{1.5(h-d_{\mathrm{c1}})} + \frac{2c}{h} \qquad \text{for } \frac{d_{\mathrm{c1}}}{h} > 0.15 \end{aligned} \tag{3.5}$$

With notching of both flanges:

$$\alpha = \frac{h}{2(h-[d_{\mathrm{c1}}+d_{\mathrm{c2}}])} + \frac{4c}{h} \tag{3.6}$$

3.5 Beams with intermediate restraint to compression flange

A common scenario in steel construction is where one or more parallel rows of simply supported beams are braced at the level of their top flange, with the bracing acting in tension or compression to transfer the load to a stiff lateral support (e.g. a plan bracing system). In general, the function of the restraints is to prevent significant lateral deflection of the beam at the bracing points, such that the effective buckling length of the beam is based on the longitudinal distance between the restraints. To perform such a role, the restraints require a minimum strength and stiffness, as discussed in Section 2.8.

BS EN 1993-1-1 allows for the stiffness of the bracing system by considering its deflection δ_q under the equivalent stabilizing force q_d and any external loads. If the lateral support can be regarded as rigid as shown in Figure 3.7(a) and (b), then the deflection of the restraints is solely a function of the axial stiffness of the struts/ties. In cases where the bracing action is provided by a linear system of components,

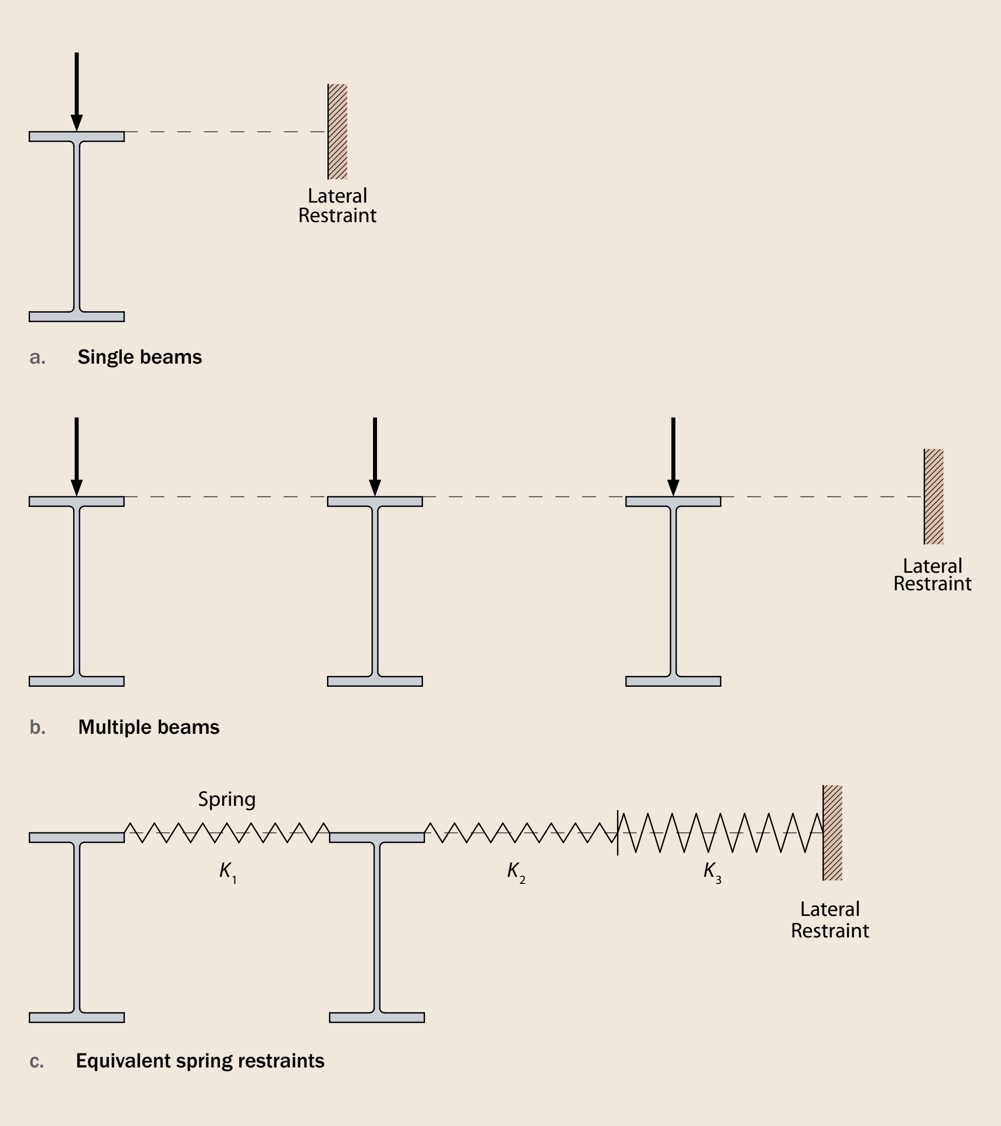

Lateral Restraint
a. Single beams
Lateral Restraint
b. Multiple beams
Spring
K_1
K_2
K_3
Lateral Restraint
c. Equivalent spring restraints

Figure 3.7 Intermediate restraints to single or multiple beams

represented by springs acting in series as shown in Figure 3.7(c), then the combined deflection of the bracing system δ_q at a section should be obtained from:

$$\delta_q = \delta_{q1} + \delta_{q2} + \delta_{q3} + \ldots$$

where

δ_{q1}, δ_{q2}, δ_{q3} etc. are the individual in-plane deflections of the components in the system.

3.5.1 Single beams

The following guidance, based on Clause 5.3.3 of BS EN 1993-1-1, is given for single beams.

The design force for a single brace at any location = $q_d L$, plus any additional forces due to external actions, where q_d is the equivalent stabilizing force and L is the length of the restrained beam (see also Section 2.8).

When more than one brace along the length of the beam is used, each should be designed to resist a force of not less than $5q_d L/8$, plus any additional forces due to external actions.

Determination of restraint forces is, in general, an iterative process, due to the dependence of the forces on the level of deflection of the bracing system. This process is described in Section 2.8 and illustrated in Worked Example 3. However, the deflections of typical bracing systems in buildings are unlikely to exceed $L/2000$. If this value is assumed for δ_q, restraint forces of 2.0% of the maximum design force in the compression flange of the beam for a single restraint emerge from Equation (2.41) (for two or more braces, 1.25% for each restraint), to which any additional forces due to external actions should be added. No iteration is required, provided that the actual deflection of the bracing system δ_q (under the above restraint forces plus any additional forces due to external actions) is less than $L/2000$. Should δ_q exceed $L/2000$, then the obtained restraint forces will be unsafe and iteration, by substituting the actual value of δ_q into Equation (2.41), will be required.

If the above conditions are met, the beam may be designed for an effective length L_{cr} equal to the distance between braced points. The bracing members not meeting these conditions should be considered to be ineffective.

The lateral support must be capable of safely withstanding the bracing force transferred to it.

Where there is a splice in a beam, the bracing system should be able to resist the effects of a lateral force equal to 1% of the design force in the compression flange at the splice location.

3.5.2 Multiple beams

For restraints to multiple beams, the following guidance is given.

The bracing should be designed for the sum of the restraint forces for the individual beams, which are determined as in Section 3.5.1 but with a reduced imperfection defined by α_m (Equation (2.43)).

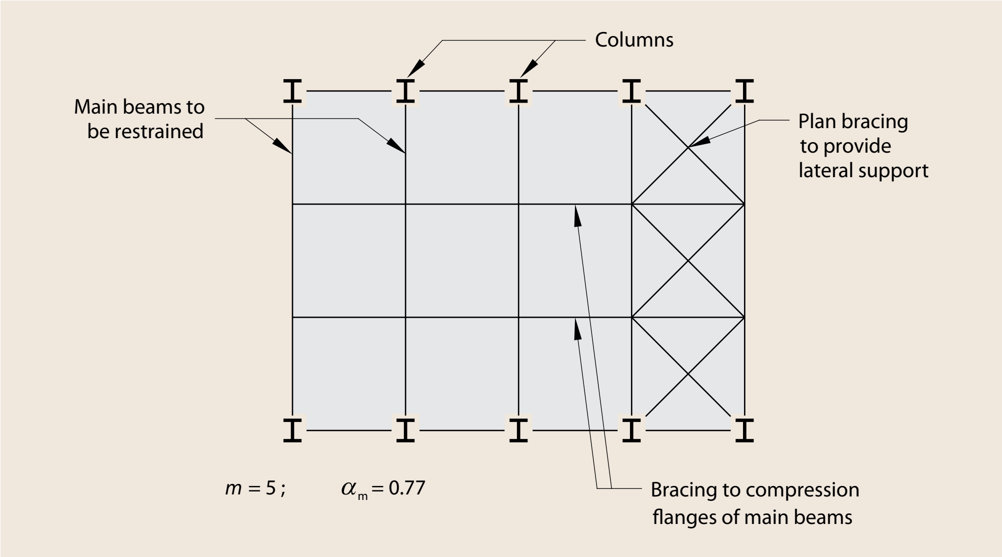

Columns
Main beams to be restrained
Plan bracing to provide lateral support
$m = 5$;
$\alpha_m = 0.77$
Bracing to compression flanges of main beams

Figure 3.8 Plan bracing

As a general rule, the lateral stiffness of the restraining system should exceed 25 times the combined lateral stiffness of the beams to be supported (where the stiffness of each beam is taken as $48EI/L^3$).

The restraining system is typically in the form of truss-type bracing in the plane of the compression flanges of the beams (i.e. plan bracing) – see Figure 3.8. The in-plane trusses transfer the restraint forces back to the supports of the beams.

Different forms of restraint to multiple beams are considered in Section 3.7.

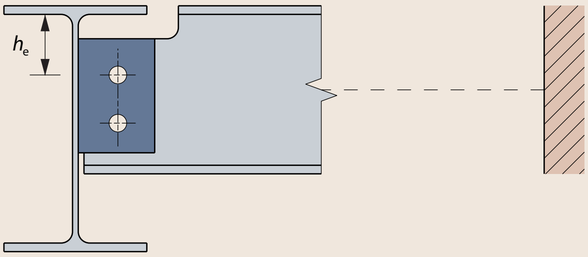
Determination of restraint forces in a bracing system for multiple beams is illustrated in Worked Example 3.

3.6 Beams with discrete intermediate restraints below top flange

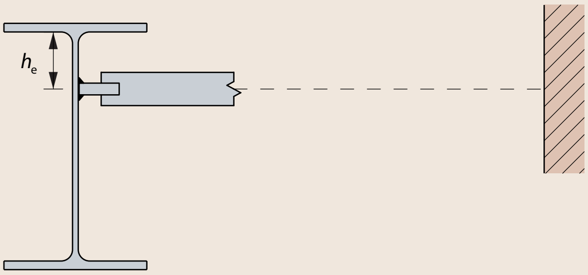
There are a number of common structural arrangements in which beams are laterally restrained below the level of the top (compression) flange. Two such arrangements with discrete intermediate restraints are illustrated in Figure 3.9. In both cases, the distance of the lateral restraint below the top flange (i.e. the exposed section height) is labelled h_e. An assessment must be made of the maximum value of h_e above which the top flange is considered as not fully restrained.

No specific guidance on the cases illustrated in Figure 3.9 is given in BS EN 1993-1-1, though the recommendations given below may be used in conjunction with the general provisions of BS EN 1993-1-1.

Connections to a beam web through web cleats or fin plates aligned vertically, as shown in Figure 3.9(a), provide effective lateral and torsional restraint. Provided the exposed web height h_e is not excessive ($h_e/t_w \leq 27$ for $f_y = 275$ N/mm^2 and $h_e/t_w \leq 22$ for $f_y = 355$ N/mm^2), full lateral restraint may be assumed at the connection points and member slenderness may be based on the distance between the connected points.



a. Discrete lateral and torsional restraint below top flange



b. Discrete lateral restraint below top flange

Figure 3.9 Beam with discrete restraint below top flange

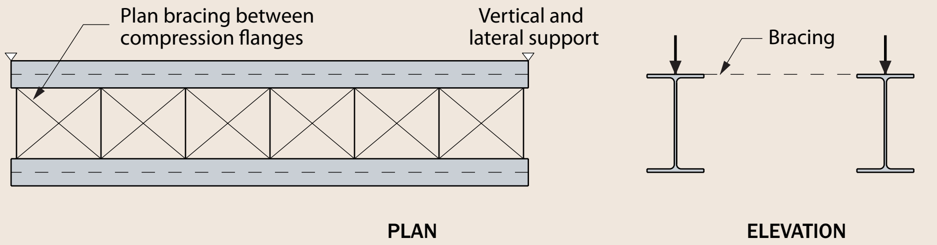
Connections to a beam web through horizontal plates, as shown in Figure 3.9(b), may offer lateral restraint, but are unlikely to offer significant torsional restraint. To be effective, such restraint should be located as close as possible to the compression flange of the beam and at least within 20% of the section height.

3.7 Braced pairs of beams

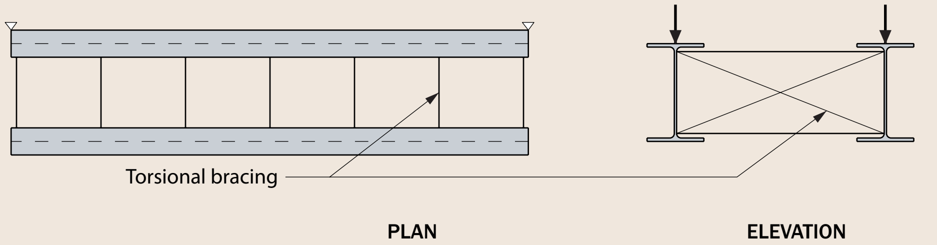
The lateral stability of two or more beams may be improved by connecting the beams together at intervals along their length by a system of plan bracing as shown in Figure 3.10(a) or, if the bracing is not triangulated in plan, with individual triangulated braces of the form shown in Figure 3.10(b). To be effective, any system of restraints that relies solely on the linking together of a series of potentially equally unstable members must ensure that instability will involve actual deformation of the bracing system. Thus merely connecting the pair of beams with a light cross-member pinned at its ends will not result in any benefit as it does not affect the buckling mode of the beams.

From Clause 5.3.3 of BS EN 1993-1-1 and with reference to Section 3.5, the following guidance is offered.

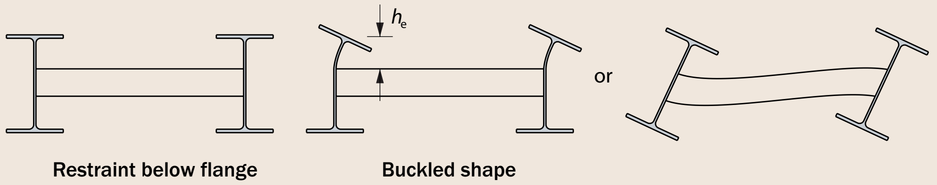
For plan bracing (Figure 3.10(a)), full lateral restraint to the compression flanges is achieved at the braced points, provided the bracing members are attached to the flanges and are capable of resisting the equivalent stabilizing force q_d (applied laterally as a uniformly distributed load to the beam). Resistance to lateral torsional buckling of the beams should be verified between the braced points.


Plan bracing between compression flanges
Vertical and lateral support
Bracing
PLAN
ELEVATION

a. **Plan bracing**


Torsional bracing
PLAN
ELEVATION

b. **Torsional bracing**


h_e
or
Restraint below flange
Buckled shape

c. **Cross-beams**

Figure 3.10 Braced pairs of beams

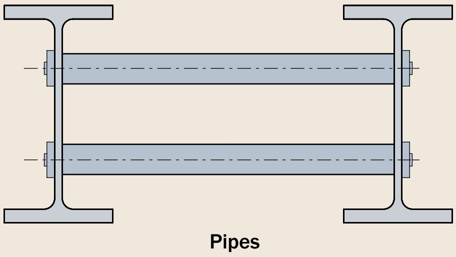
The individual restraint forces, expressed in Section 3.5 in terms of percentages of the maximum compression flange forces, are also applicable to this scenario.

For torsional bracing (Figure 3.10(b), the effective length is not limited to the distance between bracings because overall instability of the stiffened system, involving rotation of the paired beams, can occur [34]. In the absence of explicit guidance in Eurocode 3, the method set out in Section 8 of PD 6695-2 [20] may be applied.

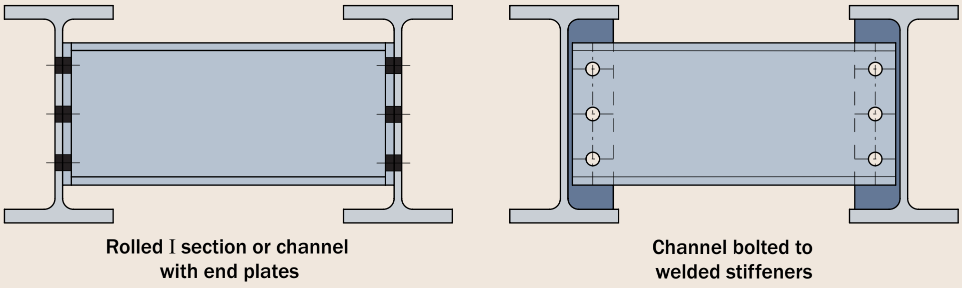
For the case shown in Figure 3.10(c)), torsional restraint is provided by means of the flexural stiffness of the cross-beams. Again, the method set out in Section 8 of PD 6695-2 may be applied. However, this arrangement will typically be less stiff than that shown in Figure 3.10(b) and will hence result in greater effective lengths and reduced lateral torsional buckling resistance. To avoid lateral-distortional buckling, it should also be checked that the depth of the unsupported web below the compression flange does not exceed the limit given in Section 3.6.

3.8 Beams supporting cavity walls

A pair of adjacent beams, typically hot rolled I sections or channels, is frequently employed to support both leaves of a cavity wall. Such beams are often interconnected at intervals by either ‘separators’ or ‘diaphragms’.



a. **Separator**



Rolled I section or channel with end plates

Channel bolted to welded stiffeners

b. **Diaphragms**

Figure 3.11 Separators and diaphragms

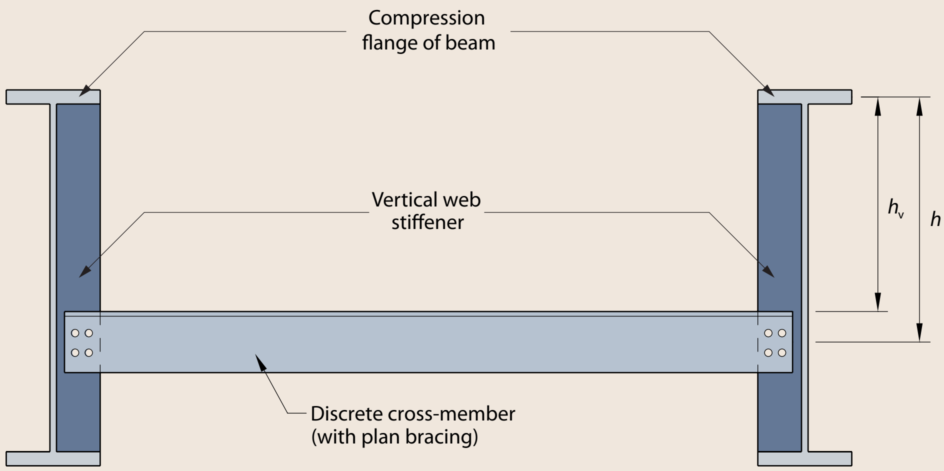
Separators act to ensure a fixed spacing between adjacent beams. They can carry transverse forces, but are not generally capable of transferring vertical forces between the beams. For equally loaded beams, separators simply dictate that both beams will buckle in the same direction, but there is no increase in load-carrying capacity beyond the sum of the individual beams.

Diaphragms are intended to retain the shape of the combined cross-section at their location and thus provide torsional restraint to the beams. Provided that they possess sufficient strength and stiffness, diaphragms may also allow effective transfer of vertical loading between the beams and their resistance may be determined as for a braced pair of beams (see Section 3.7).

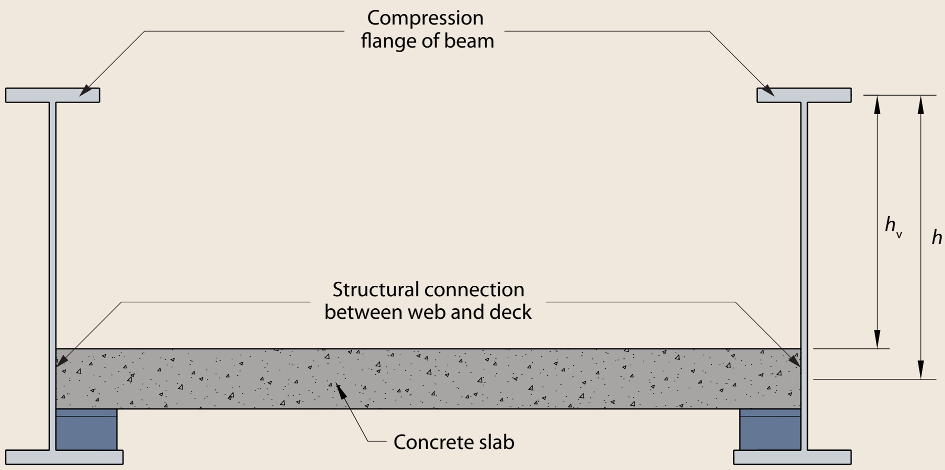
Typical examples of separators and diaphragms are shown in Figure 3.11.

3.9 Bracing by means of U-frames

Certain forms of construction, e.g. deep girder bridges, lead naturally to the type of arrangement shown in Figure 3.12(a), where a pair of parallel beams is rigidly interconnected at intervals along their length by cross-members at or just above the level of their lower flanges. The restraining effect is then transferred to the upper compression flanges, usually by means of vertical stiffeners. Stiffeners can be placed on the inside or outside of the beam, or both. In this case, the vertical stiffeners and cross-members form ‘discrete U-frames’. In the alternative arrangement of Figure 3.12(b), the cross-members are replaced by a reinforced concrete slab, and vertical stiffeners are generally not provided. In this ‘continuous U-frame’, the restraining effect is transferred to the compression flange by virtue of the stiffness of the beam web acting as a vertical cantilever.


Compression
flange of beam
Vertical web
stiffener
h_v
h
Discrete cross-member
(with plan bracing)

a. **Discrete U-frame with vertical web stiffeners and cross-members**


Compression
flange of beam
h_v
h
Structural connection
between web and deck
Concrete slab

b. **Continuous U-frame with concrete slab and unstiffened web**

Figure 3.12
U-frames

Guidance for both cases is provided in BS EN 1993-2 [35] and PD 6695-2 [20]. Further information is also given in Reference 34. The basis of the approach is to treat the compression flange and part of the compressed portion of the web as a strut on an elastic foundation, the stiffness of the 'foundation' being provided by the bending stiffness of the U-frame comprising the beam webs (with or without stiffeners) and the cross-member or concrete slab. Calculation of the U-frame stiffness is described in Section 3.9.1, while determination of the non-dimensional slenderness for lateral buckling of the compression flange is set out in Section 3.9.2.

3.9.1 U-frame stiffness

The stiffness of a discrete U-frame C_d, such as that shown in Figure 3.12(a), assuming rigid joints between its components, is presented in Table D.3 of BS EN 1993-2 as:

$$C_\mathrm{d} = \frac{EI_\mathrm{v}}{\dfrac{h_\mathrm{v}^3}{3} + \dfrac{h^2 b_\mathrm{q} I_\mathrm{v}}{2I_\mathrm{q}}} \qquad (3.7)$$

in which I_v is the second moment of area of each of the vertical stiffeners, I_q is the second moment of area of the cross-member, b_q is the spacing of the main beams, h is the distance between the centroid of the compression flange and the centroid of the cross-member, and h_v is the distance between the centroid of the compression flange and the top of the cross-member. Section properties for stiffeners should be calculated in accordance with Figure 9.1 of BS EN 1993-1-5 [36] on the basis of the stiffener width plus a length of $30\varepsilon t_w$ of web. For multiple beams, the restraint to internal beams may be determined by replacing $2I_q$ by $3I_q$ in the calculation of C_d [34].

Allowance for joint flexibility would be required if the joints between the vertical stiffeners and cross-member of the U-frame were classified as 'semi-rigid' according to Clause 5.2.2 of BS EN 1993-1-8 [37]. Guidance on joint stiffness, including generic stiffness values for typical joint details, is provided in PD 6695-2 [20].

For a continuous U-frame comprising a concrete slab and two unstiffened webs, the stiffness of the system c should be evaluated per unit length. The second moment of area of the web per unit length may be calculated as $I_v = t_w{}^3/12(1 - v^2)$, where v is Poisson's ratio. The second moment of area I_q of the concrete deck slab per unit length should be calculated on the basis of the 'cracked' concrete section, with the concrete properties transformed into equivalent steel properties by means of the modular ratio, as described in Appendix A of Reference 38.

3.9.2 Non-dimensional slenderness

The non-dimensional slenderness for lateral buckling of the compression flange, treated as a compression member, is defined in Clause 6.3.4.2(4) of BS EN 1993-2 as:

$$\bar{\lambda}_{LT} = \sqrt{\frac{A_{eff} f_y}{N_{crit}}} \tag{3.8}$$

in which A_{eff} is the sum of the compression flange area and one third of the compressed portion of the web (with effective areas being used in the case of Class 4 sections) and N_{crit} is the elastic buckling load of the same section with continuous restraint, based on gross section properties.

If the stiffness of each discrete U-frame C_d is greater than $4N_E/l$, where N_E is $\pi^2 EI/l^2$ and l is the U-frame spacing, then $\bar{\lambda}_{LT}$ may be calculated on the basis of the length between the U-frames, with $N_{crit} = \pi^2 EI/l^2$.

For flexible U-frames ($C_d < 4N_E/l$) between 'rigid' end supports (as defined by Equation (6.13) of BS EN 1993-2), N_{crit} is given in Clause 6.3.4.2(6) of BS EN 1993-2 as:

$$N_{crit} = mN_E \tag{3.9}$$

in which $N_E = \pi^2 EI/L^2$, where I is the transverse second moment of area of the effective flange and web, L is the distance between rigid end supports, and $m = 2/\pi^2\gamma^{0.5} \geq 1.0$.

The value of γ is given by cL^4/EI with c being the U-frame stiffness per unit length, taken as C_d/l for discrete U-frames spaced at a distance l. Note that N_{crit} should not be taken as larger than that of the elastic critical force for flexural buckling between discrete lateral restraints [34].

'Rigid' end supports are likely to be achieved in the case of cross-bracing, but are unlikely to be achieved simply with end U-frames [34]. Guidance on the determination of N_{crit} between non-rigid end supports is given in Section 9 of PD 6695-2 [20], in which a modified expression for m is presented.

3.9.3 Member resistance

Lateral torsional buckling resistance of the compression flange may be determined on the basis of the above non-dimensional member slenderness in conjunction with the buckling curves presented in Clause 6.3.2.2 of BS EN 1993-1-1.

The forces in the U-frame arising as a result of providing restraint against lateral buckling of the compression flange are defined in Clause 6.3.4.2 of BS EN 1993-2 and discussed in Reference 34.

3.10 Beams with tension flange restraint

The stability of beams with intermediate restraints at or adjacent to the tension flange was introduced in Section 2.7. This scenario typically arises in the outer columns and hogging moment regions of rafters in single storey portal frames, in which intermediate tension flange restraint is provided by the purlins and side rails. These elements are generally assumed to offer full lateral (translational) restraint but no torsional restraint to the tension flange of the member. Lateral restraint may be assumed to be sufficient, provided Equation BB.2 of BS EN 1993-1-1 is satisfied – see Section 3.12.

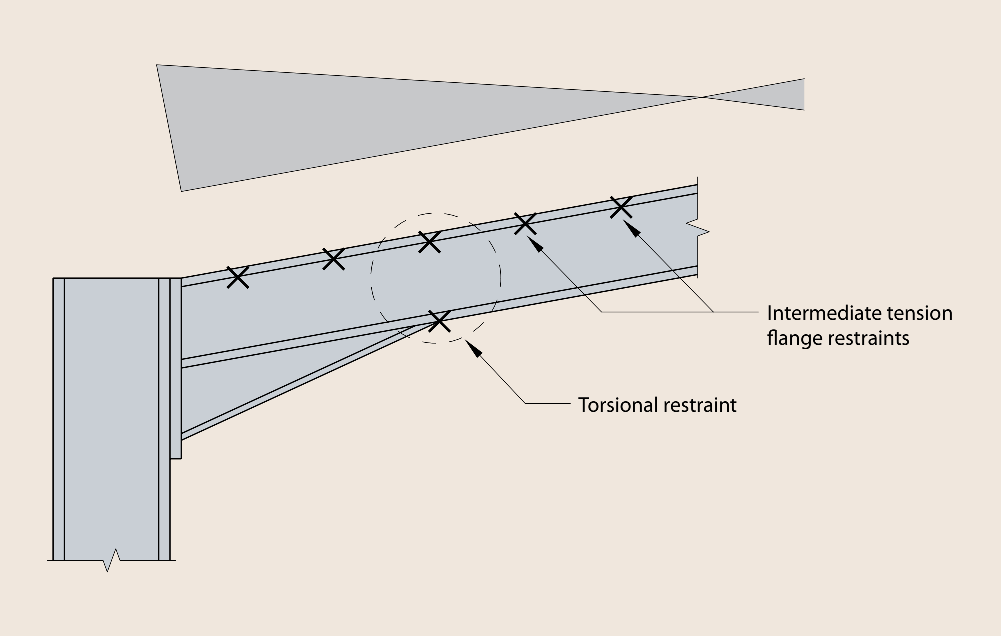


Figure 3.13 Rafter with intermediate tension flange restraint

There are two approaches that may be followed to verify the stability of the member between torsional restraints (i.e. where there is restraint to both flanges):

- Determination of member buckling resistance $M_{\mathrm{b,Rd}}$, suitable for elastic design – see Section 3.10.1.
- Determination of a stable length, generally used for plastic design – see Section 3.10.2.

For both methods, two checks are required. The first relates to the stability of the beam in a flexural torsional mode between restraints to both flanges, while the second assesses the beam stability between the intermediate tension flange restraints.

3.10.1 Member buckling resistance check

The lateral torsional buckling resistance $M_{\mathrm{b,Rd}}$ of a beam may be determined from Equation (2.15), based on the beam slenderness $\bar{\lambda}_{\mathrm{LT}}$ defined by Equation (2.18). For checking stability of the member between torsional restraints, M_{cr} is calculated as $M_{\mathrm{cr,T}}$ from Equation (2.34) for uniform moment, Equation (2.35) for a linear moment gradient or Equation (2.36) for a nonlinear moment gradient. The dimension a should be taken as the distance between the restrained longitudinal axis (e.g. the centroid of the purlins) and the shear centre of the member – see Figure 3.14. For checking stability of the member between the intermediate tension flange restraints, M_{cr} is calculated from Equation (2.33), assuming restraint to both flanges.[39]

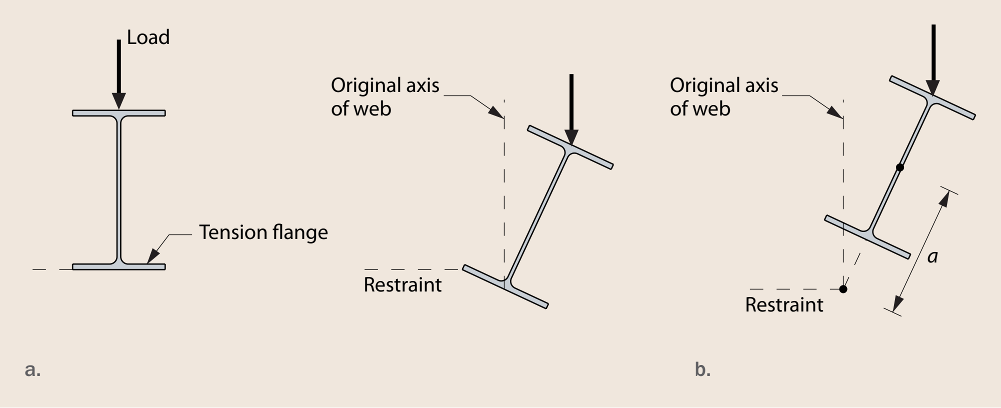


Figure 3.14 Beam with continuous tension flange restraint

3.10.2 Stable member length check

Alternatively, and for segments of members containing plastic hinges, member stability may be verified on the basis of stable member lengths, as set out in Clause BB.3.1.2 and BB.3.1.1 of BS EN 1993-1-1. Under uniform bending, the maximum stable length of a member between torsional restraints (but with intermediate tension flange restraints) is given by Equation (2.37), which is Equation BB.6 of BS EN 1993-1-1. For linear and nonlinear moment gradients, the stable length is given by Equations (2.38) and (2.39) respectively, which correspond to Equations BB.7 and BB.8 of BS EN 1993-1-1. Equation (2.40), which is Equation BB.5 of BS EN 1993-1-1, may be used to check the stable length between the intermediate tension flange restraints themselves.

3.11 Beams supporting timber floors

A series of steel beams supporting a timber floor system is a common form of construction, particularly in older buildings. A number of practical arrangements exist, in which the steel beam may be located beneath the timber joists or may lie partly or wholly within the floor depth. The degree of lateral restraint provided by the timber joists to the steel beam depends largely on the interconnection between these two components. The cases shown below are considered to be examples of good practice where there is a positive connection between the steel beam and the timber joists; in such cases, the steel beam may be assumed to be fully laterally restrained by the timber joists.

The following recommendations are made.

Positive connection between the timber joists and the steel beam is the ideal means of providing lateral restraint.

Reliance on friction alone is not generally recommended, particularly when dynamic loading is present. Lateral restraint by friction is permitted for temporary structures in BS 5975 [40], in which coefficients of friction μ between timber and steel are provided. For permanent situations, including typical residential construction, a conservative value of μ of 0.1 is recommended.

The timber plates employed in Cases (a) and (b) and the timber runners featuring in Case (d) should be bolted to the steel beam at not less than 1 m spacing.

Nails or screws should be used to provide a positive connection between the timber floor joists and timber plates in Cases (a) and (b) and between the timber floor joists and the timber runners (though the steel hangers) in Case (d).

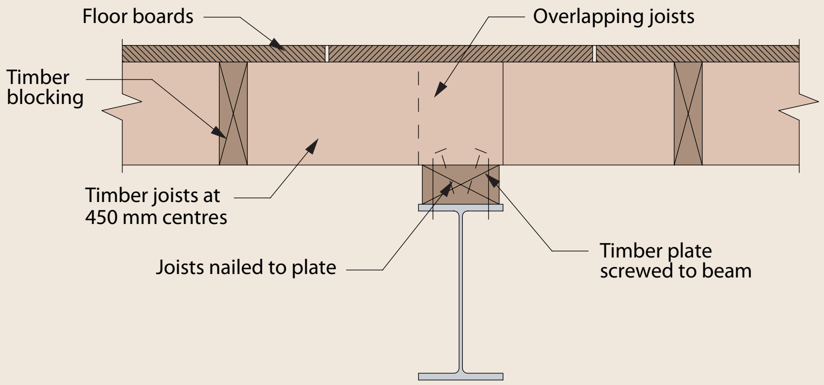
Notching of the timber joists over the top flange of the steel beam (with a tight fit) or use of timber ‘keys’ are effective in preventing lateral movement of the top flange – see Case (c). Additional steel straps are recommended for heavy beams (say $>$ 35 kg/m).

Edge beams require special consideration because notching on its own is not sufficient to provide restraint. Steel straps are generally required in this case.

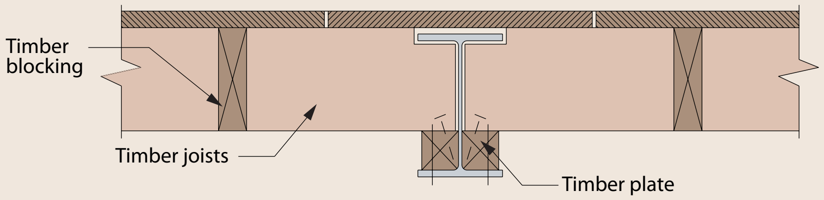
Restraint forces transferred to the timber joists are relatively small for modest sized steel beams and may be resisted either by diaphragm action of the flooring, or by packing or fixing the ends of the timber joists firmly against the masonry walls.

For larger steel beams supporting walls or other heavy loads, specific provision for lateral restraint should be made using straps and bracing, or the beam treated as discussed in Section 4.2.

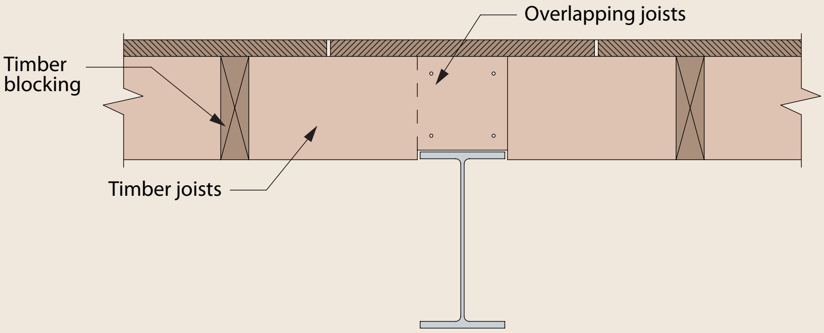
Restraint forces should be transferred to some stiff point in the structure (e.g. masonry walls) or may be carried by diaphragm action when floor boards are present and positively connected to the timber joists.


Floor boards
Overlapping joists
Timber blocking
Timber joists at 450 mm centres
Joists nailed to plate
Timber plate screwed to beam

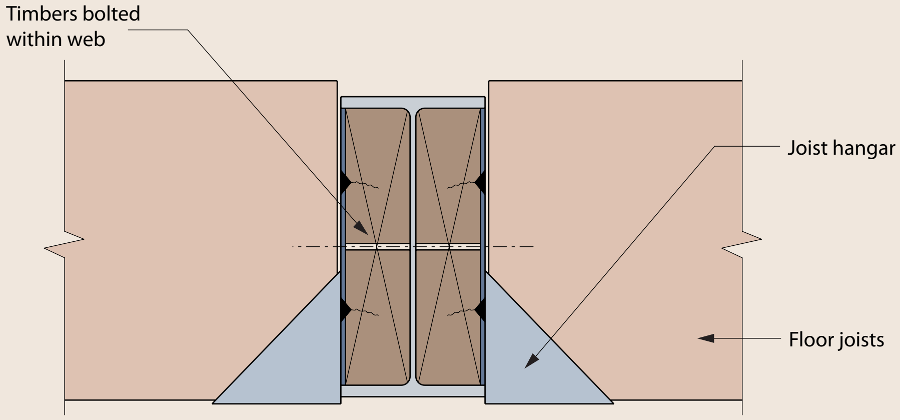
a. **Top flange bearing**


Timber blocking
Timber joists
Timber plate

b. **Bottom flange bearing**


Overlapping joists
Timber blocking
Timber joists

c. **Top flange bearing - notched joists**


Timbers bolted within web
Joist hangar
Floor joists

d. **Joist hangers**

Figure 3.15
Beams supporting timber floors

3.12 Beams supporting steel decking (or roof sheeting)

Profiled steel decking, either in the form of roof sheeting or as formwork to a composite floor, spans between the beam top flanges and is fastened with screws or pins. In composite construction this is a temporary condition developed prior to and during concreting. Two cases are considered – the first, covered in Section 3.12.1, where the decking spans perpendicular to the beam under consideration (see Figure 3.16), and the second, covered in Section 3.12.2, where the decking spans parallel to the beam under consideration (see Figure 3.17).

3.12.1 Decking spanning perpendicular to beam

In this configuration – see Figure 3.16 – the shear stiffness of the sheeting provides resistance to lateral deflections of the beam. This will typically be sufficient to provide effective continuous lateral restraint to the beam.

To verify the ability of the sheeting to provide full lateral restraint, the shear stiffness S of the sheeting (per unit of beam length) should satisfy the following criterion, given in BS EN 1993-1-1, (BB.2):

$$S \geq \frac{70}{h^2}\left(EI_{\mathrm{w}}\frac{\pi^2}{L^2} + GI_{\mathrm{T}} + 0.25h^2 EI_{\mathrm{z}}\frac{\pi^2}{L^2}\right) \qquad (3.10)$$

in which L is the length of the beam and h is the depth of the beam.

This criterion assumes that the sheeting is connected to the beam at every rib. If the sheeting is connected to the beam at every second rib, the shear stiffness required increases by a factor of five (Clause BB.2.1(1)B of BS EN 1993-1-1).

Note that the adequacy of connection for this purpose is not stated in BB.2.1. Shear studs through-deck welded will provide adequate connection. Where other means of fixing is used, it should be designed to resist restraint forces given by Section 2.8.

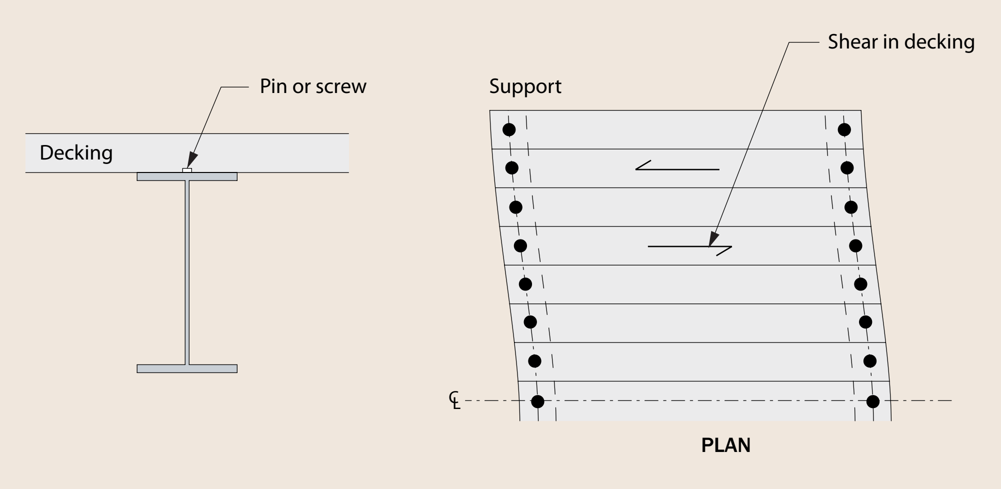


Figure 3.16 Decking spanning perpendicular to beam

The shear stiffness per unit length of trapezoidal sheeting may be calculated from Clause 10.1.1(10) of BS EN 1993-1-3[4], using the following expression:

$$S = 1000\sqrt{t^3}(50+10\sqrt[3]{b_\mathrm{r}})\frac{s}{h_\mathrm{w}} \qquad \text{(N)} \qquad (3.11)$$

in which t is the thickness of the sheeting in mm, b_r is the overall length of sheeting perpendicular to the beams (i.e. the depth of the diaphragm) in mm, s is the spacing of the members supporting the sheeting and h_w is the depth of the sheeting in mm.

Use of Equations (3.10) and (3.11) is demonstrated in Worked Example 4.

For cases where the stiffness of the sheeting is less than that required to provide full lateral restraint, the beam must be verified for resistance to lateral torsional buckling. The non-dimensional slenderness of a beam attached to profiled steel decking may be determined from Equation (3.12)[41].

$$\bar{\lambda}_\mathrm{LT} = 0.6\sqrt{\frac{n_\mathrm{p} A f_\mathrm{y} h_\mathrm{w}^{1.5} p \xi}{E t^{2.5} b_\mathrm{r}^2}} \qquad (3.12)$$

where n_p is the number of supporting beams, A is the cross-sectional area of a single beam, h_w is the depth of the profiled sheeting, p is the profile pitch, t is the thickness of the sheeting and ξ is a factor taken as 5 for fasteners in every trough and 15 for fasteners in alternate troughs.

All beams are fully laterally restrained in the composite stage (see Section 3.13.1).

3.12.2 Decking spanning parallel to beam

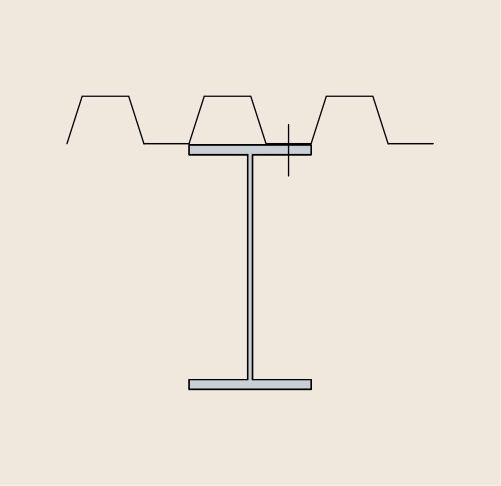


Figure 3.17 Decking spanning parallel to beam

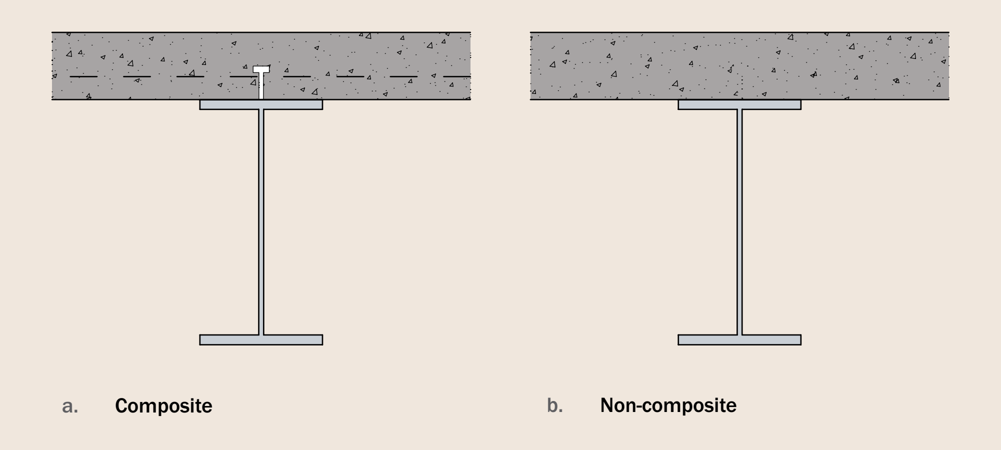
In this configuration – see Figure 3.17 – the sheeting offers little resistance to lateral deflection of the beam.

The steel beam is not restrained by the decking and its effective length is determined from the distance between the beam-to-beam or beam-to-column connections.

At the composite stage, all beams are fully restrained laterally (see Section 3.13.1).

3.13 Beams supporting concrete slabs

A system of parallel beams supporting a concrete slab may act either compositely, due to the presence of shear connectors (see Figure 3.18(a)), or non-compositely (see Figure 3.18(b)). In this section, it is assumed that joints between precast units



a. Composite b. Non-composite

Figure 3.18 Beams supporting concrete slabs

are grouted; the situation in which only the precast slabs are present (e.g. during construction) is covered in Section 3.14.

3.13.1 Composite beams

In the case of composite beams, full restraint will be provided to the beam by diaphragm action in the slab, transferred through the shear connectors. Composite beams are therefore fully restrained under imposed load. During construction, restraint may be provided by the steel decking and reference should be made to Section 3.12.

3.13.2 Non-composite beams

In the case of non-composite beams, with no positive restraint to the compression flange, lateral buckling is resisted only by friction between the steel beam and the concrete slab.

Beams may be designed to be fully laterally restrained provided that the frictional resistance per unit length of beam is greater than the required stabilizing force for one beam per unit length q_d, which may be obtained from Clause 5.3.3(2) of BS EN 1993-1-1 or from Equation (2.41). With a partial factor of 1.5 applied to the load, this leads to the following requirement:

$$w_1\mu > 1.5q_\mathrm{d} = 12N_\mathrm{Ed}\frac{e_0+\delta_\mathrm{q}}{L^2}$$

where

w_1	is the uniformly distributed design loading acting on the beam per unit length
μ	is the coefficient of friction between the steel beam and concrete slab
N_Ed	$= M_\mathrm{Ed}/h$
M_Ed	is the maximum moment in the beam and h is the overall beam depth
e_0	$= L/500$ (with no reduction for multiple beams since the stabilizing effect of friction is concerned with each beam and associated concrete slab)
δ_q	is the in-plane deflection of the bracing system (i.e. the concrete slab) due to q_d plus any external loads
L	is the length of the beam.

The in-plane deflection δ_q may be assumed to be insignificant in this scenario, due to the high in-plane stiffness of the concrete slab. Hence, for a uniformly distributed load, the above requirement reduces to:

$$w_1 \mu > \frac{12 N_{\mathrm{Ed}}}{500 L} = \frac{3 w_1 L}{1000 h}$$

For $L/h \leq 30$, the required coefficient of friction μ is about 0.1. Available test data [42,43] suggests that the average coefficient of friction between steel and concrete is greater than 0.4, with a lower bound of about 0.2. For beams with painted top flanges, μ may be smaller, but should exceed 0.1. Beams supporting concrete slabs (with grouted joints if precast units are employed) may therefore be designed as fully laterally restrained. Where the above requirement is not satisfied, the approach of Section 3.14 may be adopted.

3.14 Beams supporting precast concrete slabs

In Section 3.13, forces transferred to the concrete slab, either via the shear studs or by friction, due to lateral buckling of the steel beam are assumed to be resisted by diaphragm action in the concrete. A system of precast concrete slabs will act similarly once the joints between the units are grouted and/or a top screed is in place. However, prior to grouting (e.g. during construction) or where reliance is not put on the grout between the units, such resistance will be lost, due to the lack of longitudinal shear transfer between the individual precast units (i.e. the units can translate longitudinally relative to one another, as shown in Figure 3.19).

Lateral stability is therefore reliant on the restoring moment that develops as the beam buckles. Two cases are considered: the first relates to internal beams with symmetrical loading (see Figure 3.20(a) and Section 3.14.1) while the second considers edge beams or beams with non-symmetrical loading (see Figure 3.20(b) and Section 3.14.2). The precast concrete slabs are shown as supported on the top flange of the steel beam, but the guidance given in Sections 3.14.1 and 3.14.2 applies equally to slabs supported on shelf angles.

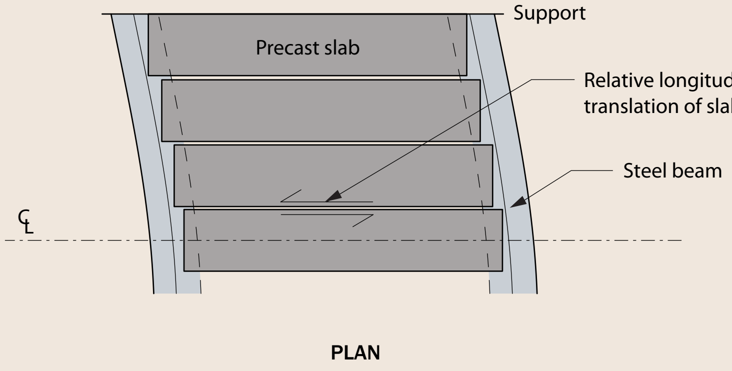


Figure 3.19
Lack of longitudinal shear transfer between precast units

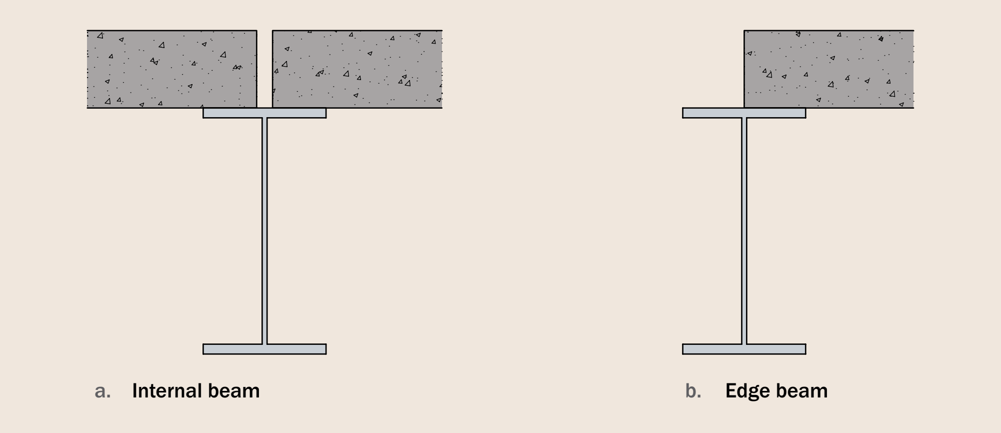


Figure 3.20 Beams supporting precast concrete slabs

3.14.1 Internal beams with symmetrical loading

Figure 3.21 shows an internal beam with symmetrical loading in the buckled configuration. As the beam buckles, the points of action of the slabs on either side of the beam shift, such that a restoring moment is generated that opposes the buckling deformation. For symmetrical loading, the restoring moment per unit length M_{Res} may be shown to be:

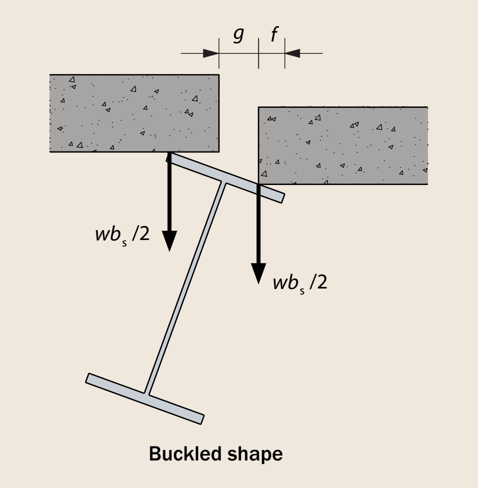


Figure 3.21 Buckling of beam supporting precast units

$$M_{\mathrm{Res}} = \frac{wb_s f}{2}$$

where w is the load per unit area from the slabs, b_s is the spacing between parallel beams and f is the width of flange covered by each slab, as shown in Figure 3.21.

The following recommendations are made for internal beams with symmetrical loading:

- Good practice requires that $f \geq 60$ mm for adequate bearing and $g \geq 20$ mm for tolerance purposes, where g is the spacing between the slabs (see Figure 3.21), leading to a minimum flange width for this form of construction of about 140 mm.
- Assuming, with a partial factor of 1.5 on loads, that the required stabilizing moment per unit length is $1.5q_d h = 3wb_s L/1000$, where q_d is given by equation (2.41), h is the depth of the beam and L is the beam length. Equating this with the restoring moment per unit length gives, for stability:

  $L \leq 500f/3$

  or

  $L \leq 10.0$ m for $f = 60$ mm

  Allowing for some dimensional deviations, the criterion becomes $L \leq 8$ m.

Therefore, in practice, medium span beams (up to 8 m span) may be assumed to be fully laterally restrained by the precast units provided the minimum bearing length is achieved. The greater the bearing length the greater the restoring moment.

Primary beams that do not receive direct loading from the precast slabs are restrained only at the beam to beam connections.

3.14.2 Edge beams or beams with non-symmetrical loading

For edge beams or beams with non-symmetrical loading, the restoring moment will be reduced and may be absent. Hence lateral restraint may not be provided. Also, it may be necessary to consider the effects of torsion on the design of edge beams (irrespective of the requirements for lateral stability), in which case reference may be made to SCI publication P385 [44]. The following recommendations are made for edge beams.

For edge beams, the slabs must cover more than half of the flange width for any restoring moment to be generated. Equating the required stabilizing and restoring moments per unit length, and making some allowance for dimensional deviations, leads to the following requirement for stability:

$L \leq 133x$

where

x is the distance to which the precast slab extends beyond the centreline of the steel section, as shown in Figure 3.22(a), up to a maximum value of $b/2$, where b is the flange width of the section.

For the case where the slabs are supported by the entire width of the flange, as shown in Figure 3.22(b), lateral restraint is provided and torsional effects may be ignored.

Internal beams may be subjected to non-symmetrical loading either as a result of unequal precast slab widths on either side of the supporting beam or unequal loading on the slabs. With reference to Figure 3.23, where the slab widths on either side of the supporting beam are denoted b_{s1} and b_{s2} and the loading from these slabs per unit area are denoted w_1 and w_2, the following recommendation is made.

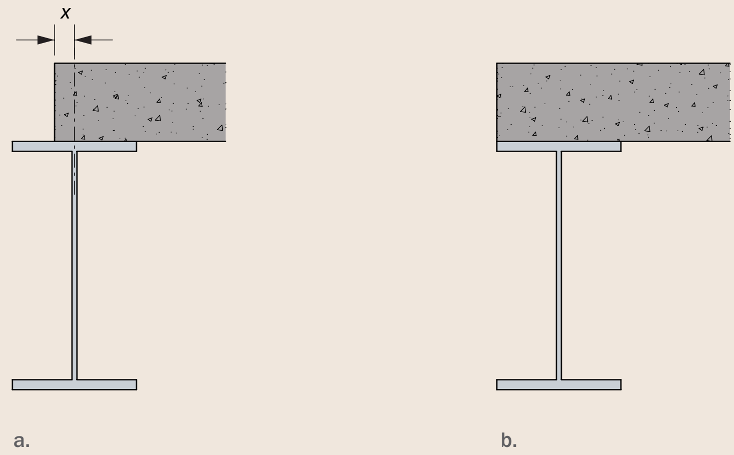


Figure 3.22 Edge beams supporting precast concrete slabs

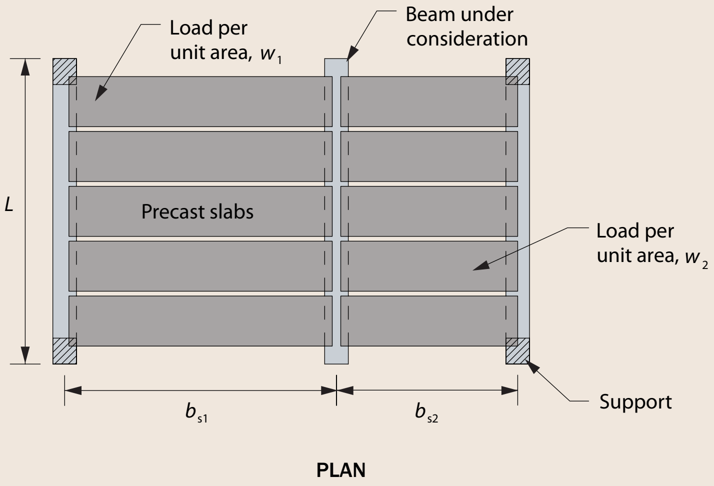

Load per
unit area, w_1
Beam under
consideration
L
Precast slabs
Load per
unit area, w_2
Support
b_{s1}
b_{s2}
PLAN

Figure 3.23
Beams with non-symmetrical loading

Equating the required stabilizing and restoring moments per unit length, and making some allowance for dimensional deviations, leads to the following requirement for stability: **(Note: see Corrigendum on next page)**

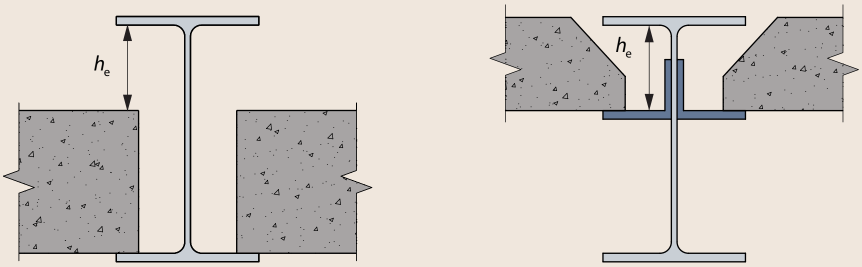
$$L \le \frac{267\left(\frac{w_2 b_2}{w_1 b_1}(f/g+0.5)-0.5\right)}{1+\frac{w_2 b_2}{w_1 b_1}}$$

The effect of combined bending and torsion should also be considered[44].

3.15 Beams with continuous restraint below top flange

Concrete slabs supported on the bottom flange of a beam (Figure 3.24(a)) or on shelf angles (Figure 3.24(b)) provide continuous restraint along the beam length, but below the level of the top flange.

No specific guidance on the cases illustrated in Figure 3.24 is given in BS EN 1993-1-1, though the recommendations given below may be employed.



a. **Concrete slab supported on bottom flange of beam**

b. **Concrete slab supported on shelf angles**

Figure 3.24
Beam with restraint below top flange

P360 – Corrigendum

(Noted March 2012)

The published expression at the end of Section 3.14.2 is as follows:

$$L \leq \frac{267\left(\dfrac{w_2 b_2}{w_1 b_1}(f/g + 0.5) - 0.5\right)}{1 + \dfrac{w_2 b_2}{w_1 b_1}}$$

This is incorrect because the RHS is dimensionless as a result of omitting the parameter 'g'.

Also, the subscripts to *b* do not match Figure 3.23.

The expression should be:

$$L \leq \frac{267g\left(\dfrac{w_2 b_{s2}}{w_1 b_{s1}}(f/g + 0.5) - 0.5\right)}{1 + \dfrac{w_2 b_{s2}}{w_1 b_{s1}}}$$

And for clarity, add:

Where *f* and *g* are defined in Figure 3.21

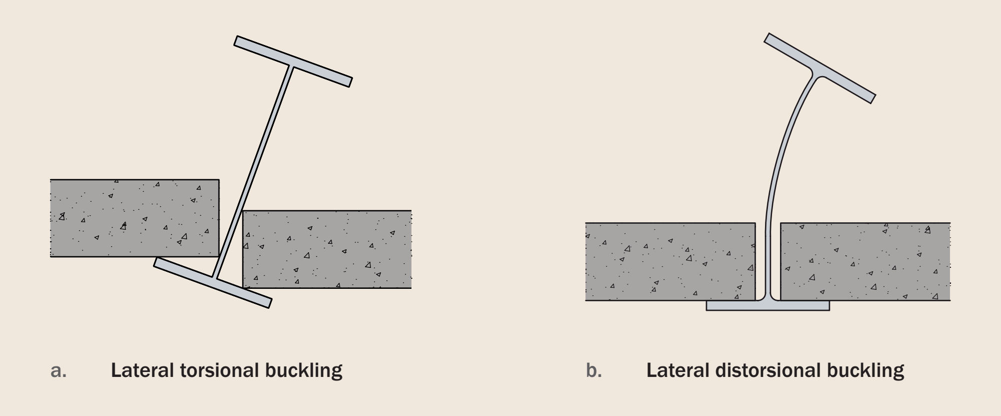


Figure 3.25
Lateral torsional and lateral distortional buckling

The possible modes of failure are lateral torsional buckling, where the cross-section remains undistorted (see Figure 3.25(a)), and lateral distortional buckling (see Figure 3.25(b)), involving flexure of the beam web.

Lateral torsional buckling

For precast construction, prior to grouting, lateral stability is reliant on the restoring moment that develops as the beam buckles. Susceptibility to lateral torsional buckling (see Figure 3.25(a)) may be assessed in the same manner as described in Section 3.14.

Edge beams do not receive significant lateral support from the adjacent precast slabs, and grouting has only a small beneficial effect. The beams are subject to combined bending and torsion[44], which often dominates the design.

Lateral distortional buckling

Once the joints between the precast units and the beam are grouted, buckling can only occur in a lateral distortional buckling mode. The key parameter then is the height of the web that is free to bend (as shown in Figure 3.25(b)).

In publication P342[46] it was shown that an approximate value of slenderness that recognises the restraint present and the distortional nature of the instability can be derived. That value may be expressed in terms of the non-dimensional slenderness as:

$$\bar{\lambda}_{\mathrm{LT}} = 0.0026\sqrt{f_{\mathrm{y}}}\,(h/t_{\mathrm{w}})^{0.75} \qquad (3.13)$$

in which f_{y} is in N/mm².

To attain the full in-plane bending resistance, $\bar{\lambda}_{\mathrm{LT}} \leq 0.4$ (for a rolled section), which requires that:

$h/t_{\mathrm{w}} \leq 27$ for $f_{\mathrm{y}} = 275$ N/mm² and $h/t_{\mathrm{w}} \leq 22$ for $f_{\mathrm{y}} = 355$ N/mm².

In the case of the situation shown in Figure 3.24(a), once the gaps between the ends of the precast units and the web are grouted, h may be taken as the exposed web height h_e (as marked in Figure 3.24(a)). In this scenario, all UKC sections when used as internal beams and supporting slabs on their bottom flanges may be designed as fully laterally restrained. For UKB sections such slabs would need to occupy between one half to two thirds of the clear web depth depending upon the beam proportions in order that the beam be designed as fully laterally restrained.

In a shelf angle floor beam (Figure 3.24(b)), lateral distortional buckling does not occur once the units are grouted.

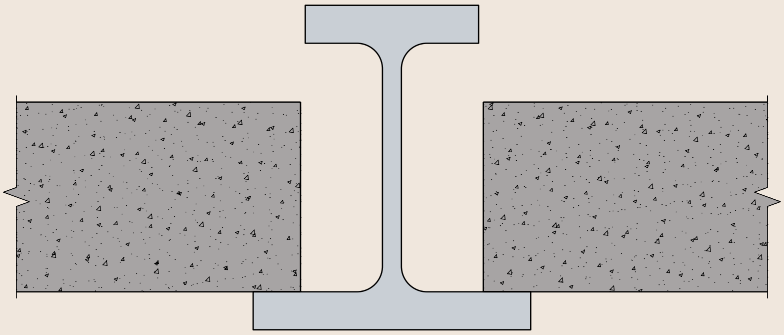
3.16 Asymmetric *Slimflor* Beams (ASBs)

Asymmetric *Slimflor* Beams (ASBs) support concrete or composite floor slabs on their wider bottom flanges. Their lateral stability, particularly during construction, requires verification. In the final condition, once the concrete has hardened, lateral torsional buckling will be prevented, as will lateral distortional buckling, provided the beam is fully encased. If the beam is only partially encased in its final condition, lateral distortional buckling, which involves flexure of the exposed web, should also be checked. ASBs with deep composite decking[45] are considered in Section 3.16.1, while those used in conjunction with precast concrete slabs[46] are addressed in Section 3.16.2. A number of design situations, including full symmetrical and non-symmetrical loading arrangements should be considered, as described in SCI publication P392[47].

3.16.1 ASBs with deep composite decking

During construction, although there is a positive connection between the composite steel decking and the bottom flange of the ASB, through the decking fixings. The top (compression flange remains unrestrained. The ASB must therefore be checked for susceptibility to lateral torsional buckling and it is recommended that the restraint given to the tension flange from the fixings is ignored. Lateral distortional buckling will not be critical during construction due to insufficient lateral restraint and will not be possible in the final condition since the ASB will be fully encased.

The lateral torsional buckling check may be conducted as described in Section 2.2 on the basis of the non-dimensional member slenderness determined through



Figue 3.26 Asymmetric Slimflor Beam (ASB) with precast slabs

Equation (2.18). The elastic buckling moment M_{cr}, must however reflect the monosymmetry of the section. Formulae for the calculation of M_{cr} are given in Reference 15.

Alternatively, and more simply, the non-dimensional member slenderness may be determined by adapting 'Method 3' in clause 6.3.2.3 of SCI publication P362 [12] based on earlier advice in SCI publication P175 [45]. For a uniformly distributed load and assuming partial restraint is provided by the end connections, the slenderness may be taken as:

$$\bar{\lambda}_{\mathrm{LT}} = 0.75V\bar{\lambda}_{z}$$

where *V*, for ASB sections, may be determined from:

$$V = \frac{1}{\sqrt{(1+0.05(\lambda_{z}/X)^{2})^{0.5}-\psi}}$$

In which $\lambda_z = L/i$, X is the torsional index (which is approximately $X = 0.8h/t_{\mathrm{f}}$ for ASB sections and is given exactly in P363), h is the overall height of the ASB and ψ is the monosymmetry index (given in P363 or conservatively may be taken as 0.6).

After construction, the ASB is fully restrained.

3.16.2 ASBs with precast concrete slabs

ASBs with precast concrete slabs should be verified for resistance to both lateral torsional buckling and lateral distortional buckling during construction.

In the construction condition before the precast units are grouted, lateral torsional buckling should be assessed in the same manner as described in Sections 3.14 and 3.15.

Once the units have been grouted, distortional buckling should be assessed, using the slenderness given by (3.13), with h taken as the height above the units. In the final condition, lateral distortional buckling is, in principle, possible if the ASB is not fully encased but in most practical situations buckling does not occur.

3.17 Composite beams in negative (hogging) bending

The top flanges of steel beams in composite sections are laterally restrained by their connection to the concrete slab, once the concrete has hardened, as discussed in Section 3.13. However, zones of composite beams in negative bending (for which the steel bottom flange is in compression) are susceptible to a form of instability that involves bending of the web, known as lateral distortional buckling – see Figure 3.27. In these zones, although the concrete slab provides continuous lateral and torsional restraint to the top flange, the unbraced bottom flange may buckle if the web has insufficient stiffness.

This problem arises at the support regions of continuous composite beams. In such cases the moment within the negative moment (hogging) region will vary as shown in Figure 3.27.

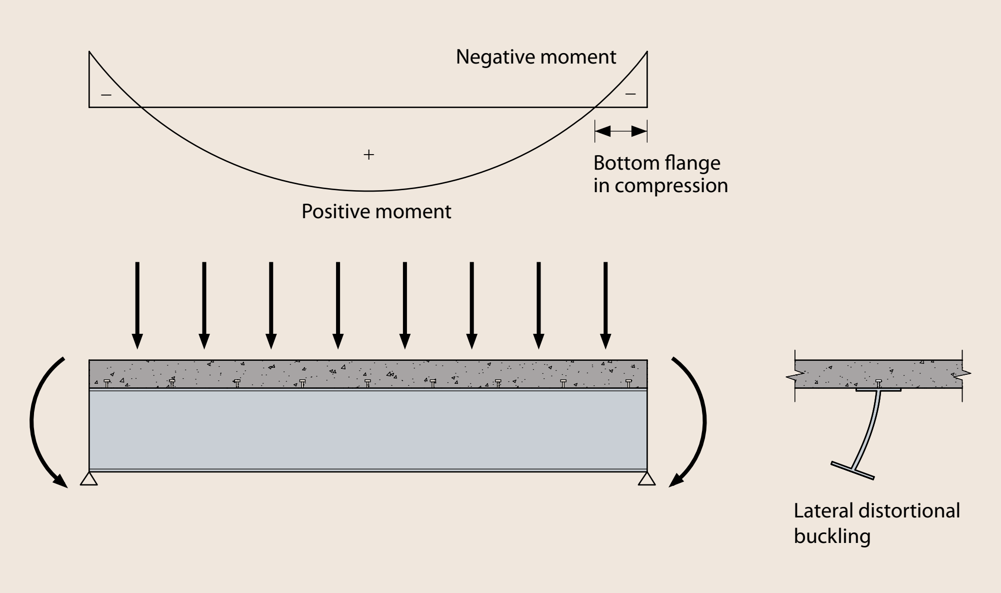


Figure 3.27 Lateral distortional buckling of composite beams in negative bending

This case may be treated as a continuous inverted U-frame in a similar fashion to that described in Section 3.8. EN 1994-1-1 offers two approaches that apply to Class 1, 2 and 3 sections – a simplified verification is set out in Clause 6.4.3 of EN 1994-1-1, while a more rigorous treatment is presented in Clause 6.4.2. In both approaches, web stiffeners must be present at the supports, but elsewhere the web should be unstiffened.

The simplified approach (Clause 6.4.3 of EN 1994-1-1) is based on identifying arrangements in which the full moment resistance may be attained without any reduction for lateral instability. Subject to a number of qualifying conditions, Table 6.1 of EN 1994-1-1 sets out limiting beam heights for IPE and HE sections, for the use of this simplified verification. Guidance for UKB and UKC sections is presented in Reference 38.

The more rigorous approach set out in Clause 6.4.2 in EN 1994-1-1 is based on the use of the lateral torsional buckling curves given in Clause 6.3.2 of BS EN 1993-1-1. Relative slenderness (the Eurocode 4 term for non-dimensional slenderness) is defined as:

$$\bar{\lambda}_{\mathrm{LT}} = \sqrt{\frac{M_{\mathrm{Rk}}}{M_{\mathrm{cr}}}} \qquad (3.14)$$

where M_{Rk} is the characteristic resistance moment of the composite section (see Section 6.2 of EN 1994-1-1), and M_{cr} is the elastic critical moment for lateral torsional buckling based on a continuous inverted U-frame. Further guidance is provided in Reference 38.

STABILIZING & DESTABILIZING LOADS

Stabilizing and destabilizing loads were introduced in Section 2.5 of this guide. In this section, practical occurrences of stabilizing and destabilizing loads are considered and design guidance is provided.

4.1 Stabilizing loads

Loads applied below the level of the shear centre of a beam that are also free to move laterally as the beam deflects, are referred to as 'stabilizing' loads. Typical examples, where the load is applied at the level of the bottom flange, are shown in Figure 4.1. As the twisting associated with buckling starts, so the line of action of the load becomes eccentric to the beam's shear centre. This induces a stabilizing effect in the form of a moment equal to $F \times e$ where e is the eccentricity of the load F.

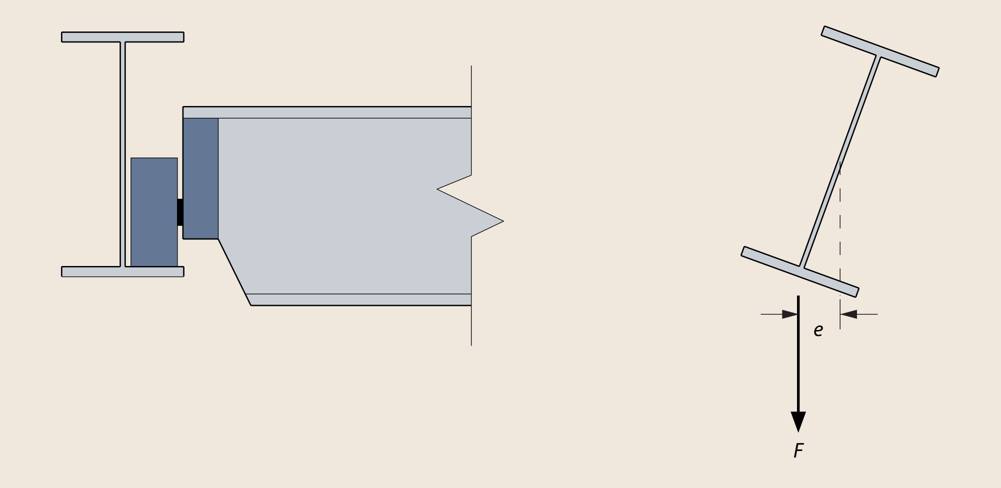


Figure 4.1
Stabilizing loads

Although this case is not covered explicitly in BS EN 1993-1-1, stabilizing loads may be allowed for either directly in the calculation of M_{cr} or more approximately through a reduction in member slenderness.

Solutions for the elastic critical buckling moment M_{cr} for particular cases may be determined from Equation (2.14) and with reference to NCCI SN003 [8], where the load level is accounted for through the z_g parameter, which is the distance between the point of load application and the shear centre of the beam. Note that z_g is negative for stabilizing loads, resulting in an increase in M_{cr} and therefore a reduction in slenderness. In practical cases of I section beams loaded at or below the bottom flange, a reduction in slenderness of up to 20% relative to a member loaded through its shear centre can result. Note that for doubly symmetric sections, the shear centre and the centroid coincide.

Although not strictly the reverse of the destabilizing case, a 10% reduction in slenderness relative to the same member loaded through its shear centre (i.e. taking $D = 0.9$ in Equation (2.21)) is a reasonable estimate to allow for the stabilizing effect of bottom flange loading.

Combined bending and torsion effects may need to be considered[44].

4.2 Beams supporting a masonry wall

Beams supporting a masonry wall either independently, or through a slab, as shown in Figure 4.2, may be considered differently depending on the level of lateral restraint provided.

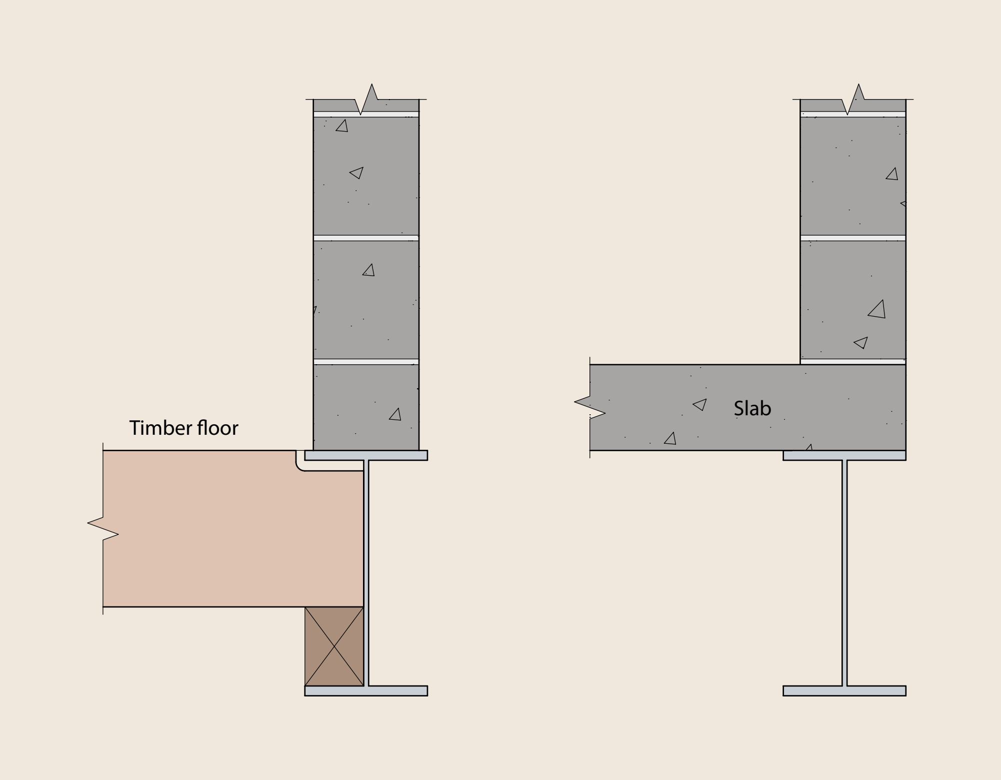


Figure 4.2
Beam supporting a masonry wall

For the case of no lateral restraint:

A conservative approach is to assume that the brickwork provides no lateral restraint and acts as a destabilizing (top flange) load. The ends of the beams are likely to be either restrained at the bottom flange only or built into a wall; suitable effective lengths for both cases are given in Sections 3.1 and 3.3, respectively.

If the line of action of the load does not pass through the shear centre of the beam, the problem is one of combined bending and torsion[44].

For the case where restraint is offered by the slab:

The top flange is prevented from displacing sideways and rotating by the connection to the slab and therefore the beam is laterally restrained, irrespective of the form of loading and the load is not destabilizing.

4.3 Destabilizing loads; beams supporting a wheel of an electrical overhead crane

In situations where a load acts at (or above) the level of the top flange of a beam and can move sideways with the beam as it starts to buckle, the load is considered to be destabilizing. This situation commonly arises in beams supporting the wheels of an electrical overhead crane, as shown in Figure 4.3. Similar examples exist in temporary works or where beams support other heavy machinery. Note that detailed design rules for crane supporting structures are provided in BS EN 1993-6[48]. In all cases, sideways movement of the load with the beam induces a destabilizing effect because the load F acts at an increasing eccentricity e to the shear centre. However, for wheels and rails of finite width it may be assumed that the load acts at the edge of the wheel or rail causing a reduction in e.

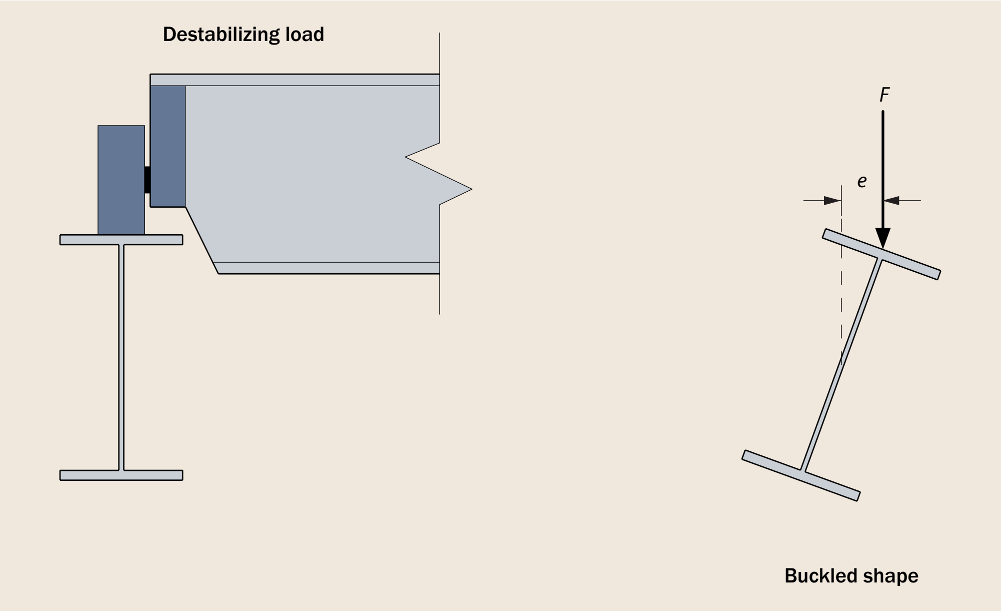


Figure 4.3
Beam supporting a wheel of an electrical overhead crane

The influence of the destabilizing load may be accounted for either in the determination of M_{cr} or through an increase in member slenderness.

The elastic critical buckling moment M_{cr} may be determined from Equation (2.14) and with reference to NCCI SN003[8], where the load level is accounted for through the z_g parameter, which is the distance between the point of load application and the shear centre of the beam. Note that z_g is positive for destabilizing loads, resulting in a reduction in M_{cr} and therefore an increase in slenderness relative to a beam loaded through its shear centre.

As a simple alternative, the destabilizing parameter D may be set equal to 1.2 in Equation (2.21) for the determination of member slenderness, which corresponds to a 20% increase in effective length. D may be determined more accurately from Equation (2.25).

For loads acting above the level of the top flange, the destabilizing parameter $D = 1.2$ may be conservatively multiplied by the ratio $2z_g/h_f$, where z_g is the distance of the point of application of the load above the beam's shear centre and h_f is the distance between the shear centres of the flanges of the beam (i.e. $h - t_f$ for an I section, where h is the overall section depth and t_f is the flange thickness). Again, D may be determined more accurately from Equation (2.25).

For a crane rail, the dimension e may be reduced on buckling as the wheel load acts only at the point of contact. In such cases, the height of the rail above the beam may be ignored (i.e. $z_g = h_f/2$ in Equation (2.14)).

LATERAL STABILITY IN PLASTIC DESIGN

This section covers the lateral stability of members in plastically designed frames, where the demands on the bracing elements are more onerous than in equivalent elastically designed frames. A review of the requirements for plastic design is presented, after which the special features and design of portal frames are discussed. Further general guidance on the plastic design of steel structures may be found in Reference 49.

5.1 Requirements for plastic design

The fundamental requirement for all members in a plastically designed frame is that they possess sufficient rotation capacity to permit the formation of the plastic hinges used as the basis for the determination of the load carrying capacity of the frame.

Since the premature occurrence of local, member or overall frame buckling will prevent this requirement from being met or lead to significant second order effects, it is necessary to consider each buckling phenomenon. Local buckling may be disregarded by using only those sections which meet the geometrical limits of Class 1 (plastic) cross-sections set out in Table 5.2 of BS EN 1993-1-1. Further cross-section requirements are given in Clause 5.6 of BS EN 1993-1-1. Overall frame buckling involving the interaction of several members in the frame may be accounted for through second order elastic-plastic analysis, or by amplifying the actions in a first order analysis using the Merchant-Rankine method, as described for portal frames in NCCI SN033[50]. Member buckling is usually taken to refer only to the out-of-plane buckling of a single member between its ends.

The basic problem for member buckling is therefore one of preventing premature lateral torsional buckling of a beam or a beam-column – guidance is set out in Clause 6.3.5 and Annex BB.3 of BS EN 1993-1-1. Clause 6.3.5.1(1)B states that lateral torsional buckling may be prevented by (1) providing effective restraints at the plastic hinge locations and (2) verifying the stable length of segments between such restraints and other lateral restraints.

Firstly therefore, effective restraints are required at points where the plastic hinges form. Where it is impractical to provide such restraint directly at the plastic hinge location, it should be provided at a distance of no more than half of the member depth away from the location (see Figure 5.1), measured along the member axis, as stated in Clause 6.3.5.2(4)B of BS EN 1993-1-1.

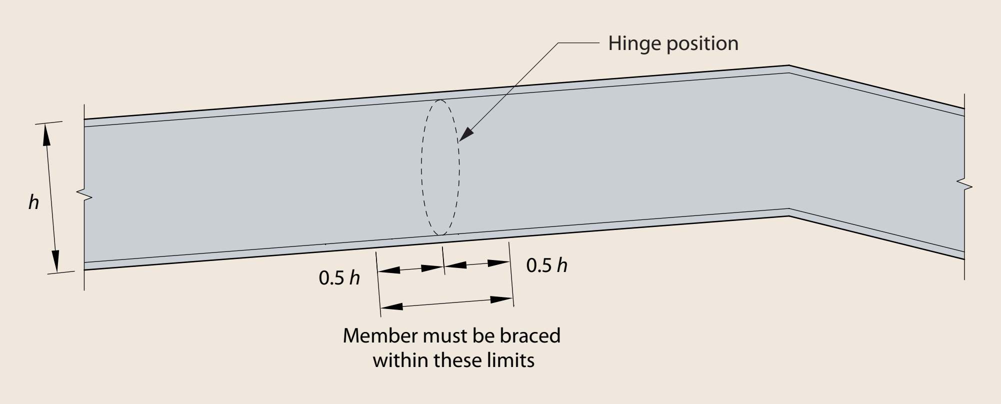


Figure 5.1 Restraint at plastic hinge locations

In this context, effective restraint is defined as lateral restraint to both flanges (Clause 6.3.5.2(2)B). This may be provided by a combination of lateral restraint at the tension flange and torsional restraint (by means of 'stays') that together restrict lateral movement of the compression flange.

Secondly, additional lateral restraints are required at a distance no further from the plastic hinge restraints than a defined 'stable length'. A simple formula for the stable length L_{stable} between compression flange restraints in a segment containing plastic hinges is given in Clause 6.3.5.3(1)B of BS EN 1993-1-1 as:

$$L_{\mathrm{stable}} = 35\varepsilon i_z \qquad \text{for } 0.625 \le \psi \le 1$$

$$L_{\mathrm{stable}} = (60 - 40\psi)\varepsilon i_z \qquad \text{for } -1 \le \psi \le 0.625 \qquad (5.1)$$

in which $\psi = M_{\mathrm{Ed,min}}/M_{\mathrm{pl,Rd}}$ is the ratio of end moments in the segment.

Equation (5.1) applies to uniform beam segments with I sections having $h/t_f \le 40\varepsilon$ and linear moment gradients. The resulting stable length L_{stable} between compression flange restraints is simple to evaluate, but does not allow for the effect of axial force and cannot be modified to take advantage of tension flange restraint.

More sophisticated formulae are given in Clause BB.3 of BS EN 1993-1-1. For a beam-column, the maximum stable length L_m between the plastic hinge and the adjacent lateral restraints is defined in Clause BB.3.1.1 of BS EN 1993-1-1 (see Equation (2.40) earlier). The equation defines the maximum distance between the plastic hinge and adjacent restraints – these restraints may be (1) compression flange restraints, (2) torsional restraints (i.e. both flanges effectively restrained) or (3) tension flange restraints, in which case a further verification on the stable length L_s (or L_k for uniform moment) to the next torsional restraint must also be performed, as described in Clause BB.3.1.2.

The stability of parts of the member outside this region (i.e. in segments not containing plastic hinges) may, of course, be checked using elastic methods by means of the interaction formulae set out in Clause 6.3.3 of this publication. The benefit of tension flange restraint may be considered as described in Section 2.7.

The demands on lateral restraints in plastically designed members are more onerous than those in elastically designed frames due to the need to stabilise members containing significant amounts of plasticity and undergoing large rotations. Design rules for restraints to members with plastic hinges are set out in Clause 6.3.5.2 of BS EN 1993-1-1, which requires:

- At each plastic hinge location, the lateral restraint must be capable of withstanding a force equal to 2.5% of the maximum force in the compression flange $N_{\mathrm{f,Ed}}$ of the braced member at the plastic hinge location.
- For the design of the bracing system, in addition to the checks for restraint forces due to member imperfections (see Clause 5.3.3 of BS EN 1993-1-1 and Sections 2.8 and 3.5 of this publication), the bracing system should also be able to resist, in combination with other loads, local forces Q_{m} applied at the plastic hinge locations of each braced member, where:

$$Q_{\mathrm{m}} = 1.5\alpha_{\mathrm{m}} \frac{N_{\mathrm{f,Ed}}}{100} \tag{5.2}$$

where α_{m} is a reduction factor, defined in Clause 5.3.3(1), based on the number of members being restrained.

It is also recommended that the non-dimensional slenderness of each restraint should not exceed 1.2 (so that it possesses adequate stiffness).

5.2 Portal frame rafters

Plastically designed portal frames represent a particular class of structure for which lateral restraints must be provided, to ensure against premature member buckling failure before developing the plastic hinge mechanism. The practicality of tying adjacent frames together to form a 3-dimensional structure will normally ensure that the column/rafter intersection is braced by so-called eaves members (or ties). Under most loading conditions, the inner flanges of the rafters will be in compression along much of their length.

Clause 6.3.5.2 of BS EN 1993-1-1 requires that all plastic hinges be laterally and torsionally restrained (i.e. restraint to both flanges). It is customary to use pairs of purlin ‘stays’ (fixed at approximately 45°) from the purlins or rails to the inner flange to provide such restraint – see Figure 5.2. The principal design problem is then to decide on the necessary locations of these stays. Normally, the proportions of the haunch are selected such that it remains elastic and hinges can potentially form in the column or in the rafter at the tip of the haunch.

The positions of the purlins and side rails are generally adjusted to be at or close to within $h/2$ of the plastic hinge locations. Stays are attached at these points between the internal flanges and these secondary members to provide effective lateral restraint to both flanges of the beam. The adjacent purlins should be placed at no

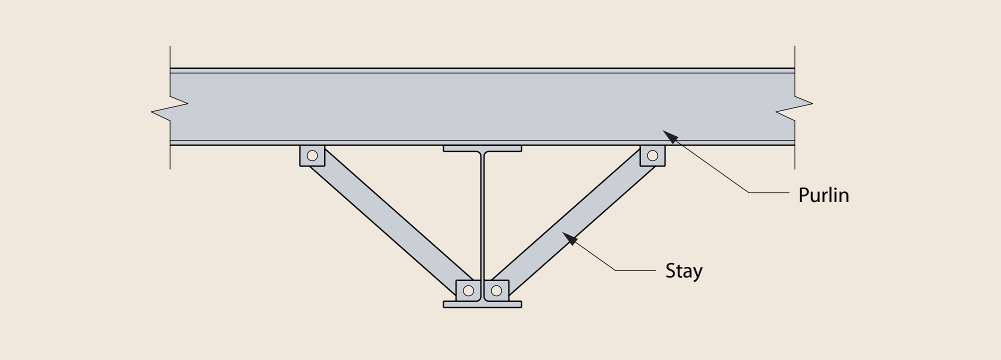


Figure 5.2 Restraint to both flanges (torsional restraint)

more than L_m (Clause BB.3.1.1 of BS EN 1993-1-1) from the plastic hinge locations, while the adjacent stays should be positioned at no more than L_s (Clause BB.3.1.2 of BS EN 1993-1-1) from the plastic hinge locations (assuming benefit is taken from the intermediate purlins providing tension flange restraint; i.e. Their spacing does not exceed L_m). The position of the stays is selected to coincide with a suitable purlin or side rail along the member length. The treatment of haunches is covered in Clause BB.3.2 of BS EN 1993-1-1.

Further design checks are then required on the elastic stability of the remaining part of the rafter or column, based on the provisions of Clause 6.3.3 of BS EN 1993-1-1. This may take account of tension flange restraint between the points of compression flange restraint, as described in Section 2.7.

COLUMNS

The design of columns is largely covered by existing guidance [15,51] and, for this reason, individual cases are not considered in detail in this Section. The principal concern is to determine the buckling length of columns correctly.

6.1 Columns in multi-storey buildings

The buckling length L_{cr} (commonly referred to as the effective length) of a compression member is the length of an otherwise similar column with 'pinned' end conditions that has the same elastic buckling load. Pinned end conditions are assumed to prevent lateral deflection of one end relative to the other but to offer no resistance to rotation.

Guidance on buckling lengths for columns in braced and unbraced frames is given below.

6.1.1 Columns in braced frames

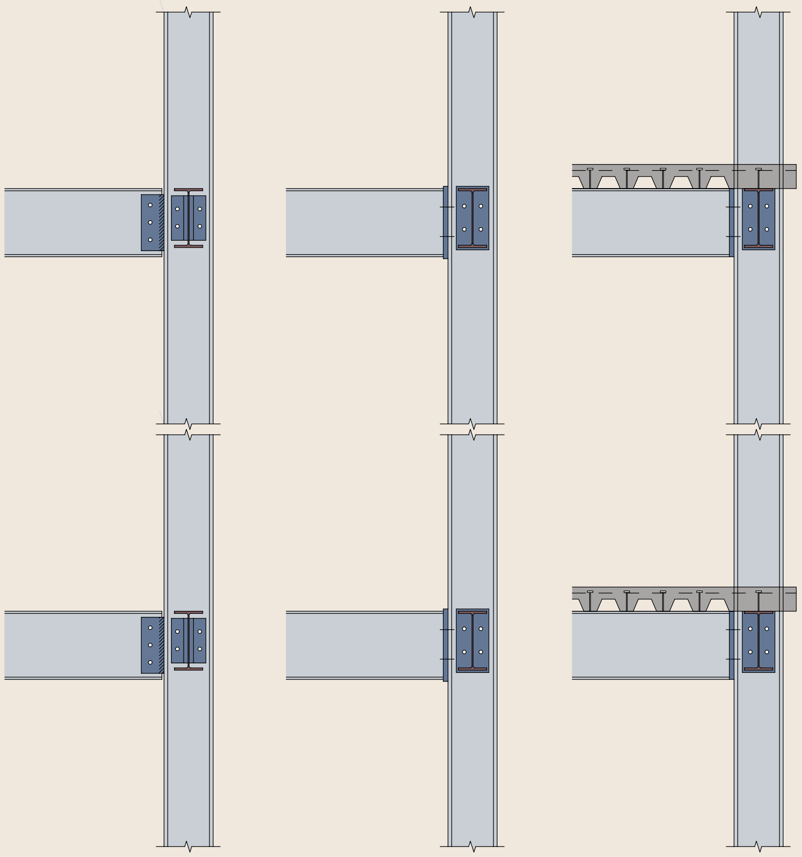
For columns in braced multi-storey frames, buckling lengths are normally taken as equal to the column lengths (i.e. $L_{cr} = 1.0L$). Lower values of buckling lengths are possible, as given in Table 6.1, when rotational restraint exists at the column ends.

The rotational end restraint afforded to columns by beam-to-column connections depends on the form of connection and whether the beams are designed compositely. Figure 6.1 shows examples of configurations in braced multi-storey frames and the appropriate buckling lengths of the columns. End plate connections are assumed to offer partial restraint. Fin plate connections should not be assumed to provide any rotational restraint.

Note that if the frame is sensitive to sway effects (i.e. $\alpha_{cr} < 10$), buckling lengths equal to the column lengths (i.e. $L_{cr} = 1.0L$) should be employed, and second order effects should be allowed for either by second order analysis or by the amplified sway method.

Table 6.1
Buckling lengths L_{cr} for columns in multi-storey braced frames

RESTRAINT (IN THE PLANE UNDER CONSIDERATION) BY OTHER PARTS OF THE STRUCTURE		L_{cr}
The column is effectively held in position at both ends	Effectively restrained against rotation at both ends	$0.7L$
	Partially restrained against rotation at both ends	$0.85L$
	Restrained against rotation at one end	$0.85L$
	Not restrained against rotation at either end	$1.0L$



	a. **Fin plate (shown in figure) or flexible end plate connections**	b. **Full depth end plate connections**	c. **Composite connections**
Internal Columns	$L_{cr} = L$	$L_{cr} = 0.85L$	$L_{cr} = 0.7L$
Edge or corner Columns	$L_{cr} = L$	$L_{cr} = 0.85L$	$L_{cr} = 0.85L$

Notes:

1. Based on minor axis buckling of the columns
2. Edge or corner columns have attachment to two or three beams respectively. Worst case is assumed to be minor axis connection to one beam
3. Beams and columns of similar stiffness

Figure 6.1 Buckling lengths of columns in non-sway multi-storey braced frames

6.1.2 Unbraced frames

In rigid jointed unbraced frames, lateral stability relies on the flexural stiffness of the connected beams and columns. For frames with $\alpha_{cr} \geq 10$, no account need be made for second order effects and buckling lengths of the columns may be taken as equal to their actual lengths (i.e. $L_{cr} = 1.0L$). For frames with $\alpha_{cr} < 10$, second order effects need to be accounted for. This may be done by second order analysis, the amplified sway method or through the use of increased (sway mode) column buckling lengths. If either the first or second of these approaches is adopted, the buckling lengths of the columns may be taken as equal to the system length. If the 'effective length method' is employed, buckling lengths greater than their system length are required, as set out in NCCI SN008[52] and Reference 15.

6.2 Columns in single storey buildings

In single storey industrial-type buildings, side rails and purlins are usually attached to the columns and beams such that the buckling lengths of the members are reduced for buckling in the minor axis (z-z) direction. It is implicitly assumed that these secondary members act as bracing points and that some longitudinal bracing is provided for overall stability. When sufficient restraints are provided, the member will buckle about the major axis (y-y) direction rather than about the minor axis. The in-plane buckling lengths of a column (in the major axis direction) in a typical industrial building are presented in Table 6.2 (taken from NCCI SN031[56]), where L refers to the column height.

Table 6.2 Effective length factors for in-plane buckling in portal frame building

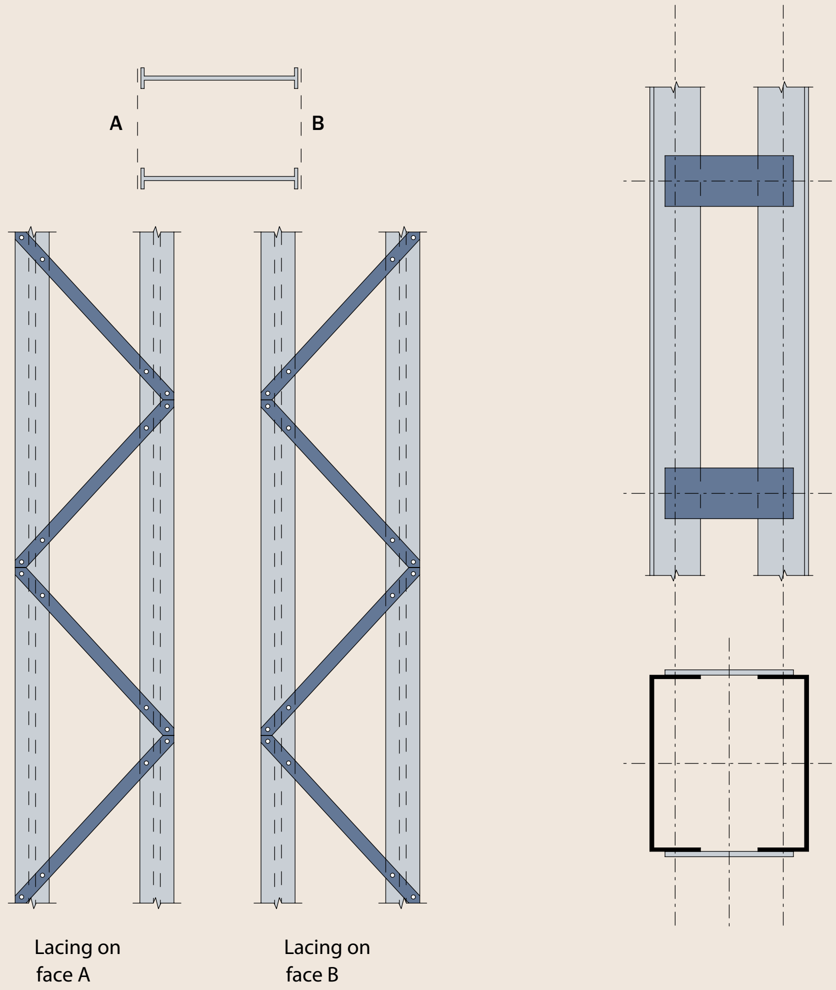
CONNECTION TO BASE	CONNECTION AT EAVES LEVEL	EFFECTIVE LENGTH FACTOR L_{cr}/L
Nominally pinned	Fixed	1.5
Fixed	Nominally pinned	1.5
Fixed	Fixed	1.2

The cases shown in Table 6.2 assume that the in-plane stiffness of the frame provides the overall stability of the building in the major axis direction. Where overall stability is provided by in-plane roof and wall bracing, the effective length of the columns may be reduced.

6.3 Built-up columns

The treatment of column buckling presented in Section 2.1 relates to the behaviour and design of rolled sections as individual members according to the rules of Clause 6.3.1 of BS EN 1993-1-1. Design rules for built-up members, consisting of two or more identical main members interconnected at intervals along their length, are presented in Clause 6.4 of BS EN 1993-1-1. The design approach is based on the analysis of an equivalent continuous member with a smeared shear stiffness S_v and an initial geometric imperfection e_0. The chord forces $N_{ch,Ed}$ are established on the basis of the applied axial load N_{Ed} (divided equally between the chord member) plus the additional axial load from the bending moment M_{Ed} induced as a result of the imperfection – see Clause 6.4.1(6) of BS EN 1993-1-1. This bending moment is calculated at the mid-height of the column as the applied axial load multiplied by the initial geometric imperfection, amplified to allow for the lateral deflection of the member. The moment amplification due to lateral deflections takes account of both the bending and shear stiffness of the built-up column.

Figure 6.2 illustrates two practical arrangements – laced and battened compressions members – which are covered in Clauses 6.4.2 and 6.4.3 of BS EN 1993-1-1, respectively. For each type of construction, checks on the interconnections (laces or



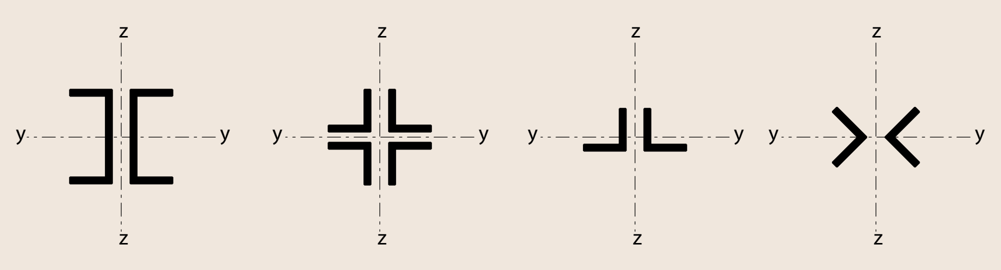
a. **Laced members**

b. **Battened members**

Figure 6.2
Built-up columns – laced and battened members

battens) should be performed for the end sections taking account of the shear force in the built-up member, as described in Clause 6.4.1(7). Further guidance is given in Reference 6.

Closely spaced built-up members, such as those shown in Figure 6.3, where the chords are either in direct contact with one another or are closely spaced and connected through packing plates, may be designed as single integral members provided that the spacing between the interconnections is within the limits set out in Table 6.9 of BS EN 1993-1-1.


z
y
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z
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y
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z
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z

Figure 6.3
Closely spaced built-up members

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CREDITS

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APPENDIX A

This Appendix presents four short examples that illustrate some aspects relating to the restraint of beams against buckling. The examples are:

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1 Beam with end restraint to bottom flange only

1.1 Introduction

This example illustrates the calculation of the design buckling resistance moment for a beam with end restraint to the bottom flange only, and considers the effect of destabilizing load. The beam is subject to a central point load and is supported at either end on masonry walls spaced 7 m apart. Two cases are considered:

Case 1: the load is suspended from the bottom flange of the beam.
Case 2: the load is applied at the level of the top flange of the beam.

Positive connection is made between the beam and the walls by bolting the bottom flange of the beam to a plate fixed on the top of the wall, as illustrated below.

The beam is a 305 × 165 × 46 UKB, grade S275, and the design value of the applied load F_{Ed} is 55 kN. (An allowance for the self-weight of the beam has been made in this value.)

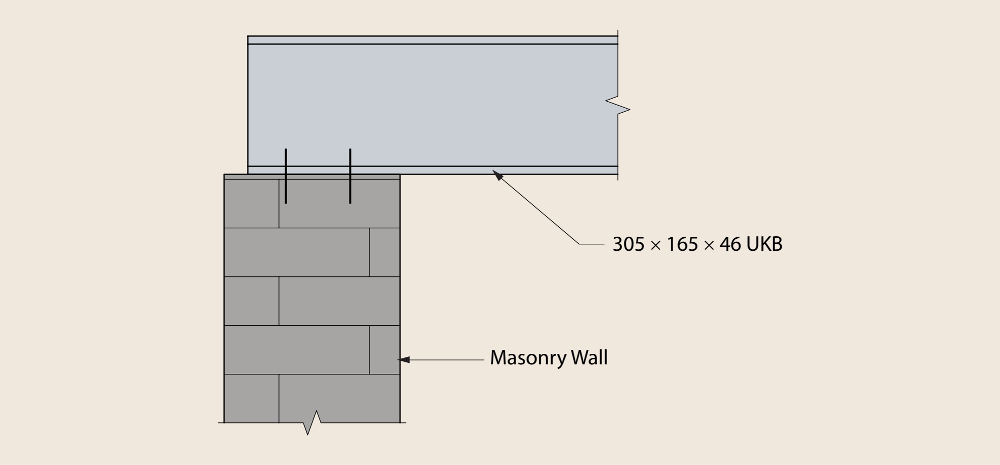

Figure 1.1
End restraint detail

1.2 Beam properties

SCI P363 Required beam properties for 305 × 165 × 46 UKB:

$h = 306.6\ \mathrm{mm}$

$b = 165.7\ \mathrm{mm}$

$W_{\mathrm{pl,y}} = 720\ \mathrm{cm}^3$

$I_z = 896\ \mathrm{cm}^4$

$I_w = 0.195\ \mathrm{dm}^6$

$I_T = 22.2\ \mathrm{cm}^4$

$t_f = 11.8\ \mathrm{mm}$

Material properties

3-1-1/3.2.1 and NA.2.4
BS EN 10025–2

$t_f \leq 16\ \mathrm{mm}$, therefore $f_y = 275\ \mathrm{N/mm}^2$

1.3 Design value of bending moment

$$M_{\mathrm{Ed}} = F_{\mathrm{Ed}}L/4 = 55.0\times 7/4 = 96.3\ \mathrm{kNm}$$

1.4 Case 1 – Load suspended from bottom flange

Section 3.3 For a beam with end restraints to the bottom flange only, under a normal (i.e. non-destabilizing) load, the effective length is taken as $kL = L + 2h$:

$$kL = L + 2h = 7000 + (2 \times 306.6) = 7610\ \mathrm{mm}$$

Section 2.2.1 On the basis of this effective length, the elastic critical moment of the beam, with $C_1 = 1.35$ for a central point load, may be calculated from Equation (2.13) in this publication:

$$M_{\mathrm{cr}} = C_1\frac{\pi}{kL}\sqrt{EI_zGI_{\mathrm{T}}}\sqrt{1+\frac{\pi^2EI_{\mathrm{w}}}{(kL)^2GI_{\mathrm{T}}}}$$

$$= 1.35\frac{\pi}{7610}\sqrt{210000\times 896\times 10^4\times 81000\times 22.2\times 10^4}$$

$$\sqrt{1+\frac{\pi^2\times 210000\times 0.195\times 10^{12}}{7610^2\times 81000\times 22.2\times 10^4}}\times 10^{-6}$$

$$= 121\ \mathrm{kNm}$$

Hence, the non-dimensional slenderness is:

$$\bar{\lambda}_{\mathrm{LT}} = \sqrt{\frac{W_{\mathrm{pl,y}}f_{\mathrm{y}}}{M_{\mathrm{cr}}}} = \sqrt{\frac{720\times 10^3\times 275}{121\times 10^6}} = 1.28$$

$$h/b = 306.6/165.7 = 1.85$$

NA.2.17 For the case of rolled and equivalent welded sections, for I sections with $h/b \leq 2$, use buckling curve ‘b’ ($\alpha = 0.34$). For rolled sections, $\beta = 0.75$ and $\lambda_{\mathrm{LT,0}} = 0.4$.

Hence

3-1-1/6.3.2.3

$$\Phi_{\mathrm{LT}} = 0.5[1+\alpha_{\mathrm{LT}}(\bar{\lambda}_{\mathrm{LT}}-0.4)+0.75\bar{\lambda}_{\mathrm{LT}}^2]$$

$$= 0.5[1+0.34(1.28-0.4)+0.75\times 1.28^2] = 1.26$$

$$\chi_{\mathrm{LT}} = \frac{1}{\Phi_{\mathrm{LT}}+\sqrt{\Phi_{\mathrm{LT}}^2-0.75\bar{\lambda}_{\mathrm{LT}}^2}} = \frac{1}{1.26+\sqrt{1.26^2-0.75\times 1.28^2}} = 0.53$$

The design buckling resistance moment is therefore given by:

3-1-1/6.3.2.1

$$M_{\mathrm{b,Rd}} = \frac{\chi_{\mathrm{LT}}W_{\mathrm{pl,y}}f_{\mathrm{y}}}{\gamma_{\mathrm{M1}}} = \frac{0.53\times 720\times 10^3\times 275}{1.0}\times 10^{-6} = 106\ \mathrm{kNm}$$

Since this is greater than M_{Ed} (96.3 kNm), the beam has sufficient resistance to lateral torsional buckling.

1.5 Case 2 – Load applied to top flange

Section 3.3 For this design situation, the load is destabilizing and thus the effective length is increased to $kL = 1.2L + 2h$:

$$kL = 1.2L + 2h = (1.2 \times 7000) + (2 \times 306.6) = 9010 \text{ mm}$$

Section 2.2.1 On the basis of this effective length, the elastic critical moment of the beam, with $C_1 = 1.35$ for a central point load, may be calculated from Equation (2.13) in this publication:

$$M_{\mathrm{cr}} = C_1 \frac{\pi}{kL}\sqrt{EI_z GI_{\mathrm{T}}}\sqrt{1+\frac{\pi^2 EI_{\mathrm{w}}}{(kL)^2 GI_{\mathrm{T}}}}$$

$$= 1.35\frac{\pi}{9010}\sqrt{210000 \times 896 \times 10^4 \times 81000 \times 22.2 \times 10^4}$$

$$\sqrt{1+\frac{\pi^2 \times 210000 \times 0.195 \times 10^{12}}{9010^2 \times 81000 \times 22.2 \times 10^4}} \times 10^{-6}$$

$$= 97.8 \text{ kNm}$$

Hence, the non-dimensional slenderness is:

$$\bar{\lambda}_{\mathrm{LT}} = \sqrt{\frac{W_{\mathrm{pl,y}} f_{\mathrm{y}}}{M_{\mathrm{cr}}}} = \sqrt{\frac{720 \times 10^3 \times 275}{97.8 \times 10^6}} = 1.42$$

NA.2.17 As for Case 1, use buckling curve ‘b’ ($\alpha = 0.34$), with $\beta = 0.75$ and $\bar{\lambda}_{\mathrm{LT,0}} = 0.4$.

Hence

3-1-1/6.3.2.3

$$\Phi_{\mathrm{LT}} = 0.5[1+\alpha_{\mathrm{LT}}(\bar{\lambda}_{\mathrm{LT}} - 0.4) + 0.75\bar{\lambda}_{\mathrm{LT}}^2]$$

$$= 0.5[1+0.34(1.42-0.4)+0.75 \times 1.42^2] = 1.43$$

$$\chi_{\mathrm{LT}} = \frac{1}{\Phi_{\mathrm{LT}} + \sqrt{\Phi_{\mathrm{LT}}^2 - 0.75\bar{\lambda}_{\mathrm{LT}}^2}} = \frac{1}{1.43+\sqrt{1.43^2 - 0.75 \times 1.42^2}} = 0.46$$

The design buckling resistance moment is therefore given by:

3-1-1/6.3.2.1

$$M_{\mathrm{b,Rd}} = \frac{\chi_{\mathrm{LT}} W_{\mathrm{pl,y}} f_{\mathrm{y}}}{\gamma_{\mathrm{M1}}} = \frac{0.46 \times 720 \times 10^3 \times 275}{1.0} \times 10^{-6} = 91.5 \text{ kNm}$$

Due to the destabilizing nature of the load, the design buckling resistance moment has reduced from 106 kNm in Case 1 to 91.5 kNm. As this value is lower than M_{Ed} (96.3 kNm), the section is now inadequate and a larger section must be used.

2 Notched beam

2.1 Introduction

Determine the design buckling resistance of a 5.0 m long unrestrained secondary beam subjected to a uniformly distributed load at the level of the top flange and supported at its ends by primary beams. The primary beams are 457 × 191 × 82 UKBs and the secondary beam is a 356 × 127 × 33 UKB. All beams are grade S275.

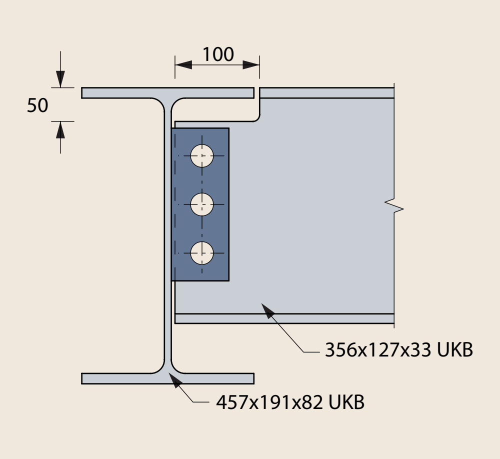

Figure 2.1
Connection details

The secondary beam is notched at its top flange, as shown in Figure 2.1, to accommodate the connection, following the recommendations given in SCI Publication P358. Verify that the notched region meets local stability criteria and determine its buckling resistance moment.

2.2 Beam properties

SCI P363

Section properties for 356 × 127 × 33 UKB:

$h \quad = 349.0\text{ mm}$

$b \quad = 125.4\text{ mm}$

$L \quad = 5.0\text{ m}$

$t_f \quad = 8.5\text{ mm}$

$t_w \quad = 6.0\text{ mm}$

$W_{pl,y} \quad = 543\text{ cm}^3$

$I_z \quad = 280\text{ cm}^4$

$I_w \quad = 0.0812\text{ dm}^6$

$I_T \quad = 8.79\text{ cm}^4$

Material properties:

3-1-1/3.2.1 & NA.2.4, BS EN 10025-2

$t_f \leq 16$ mm, therefore $f_y = 275\text{ N/mm}^2$

The dimensions of the notch are:

$d_{c1} \quad = 50\text{ mm}$

$c \quad = 100\text{ mm}$

2.3 Local stability check

$d_{c1} \quad = 50\text{ mm} < h/2 = 128\text{ mm}$

$h/t_w \quad = 349/6 = 58.2 > 55.1$ (for S275), so verify:

Section 3.4.1

$$c \le \frac{167000h}{(h/t_{\mathrm{w}})^3} = \frac{167000 \times 349}{(55.1)^3} = 348 \text{ mm}$$

Since $c = 100 \text{ mm} < 348.4 \text{ mm}$, the notched beam is OK with regard to local stability in the notched region.

2.4 Buckling resistance moment

The procedure to check for overall stability is to calculate the elastic critical moment normally and then apply a reduction factor, r_{n}, to determine $M_{\mathrm{cr,n}}$ for the notched beam. Overall stability can then be checked using this critical moment value.

Section 2.2.1 The elastic critical moment for an unnotched beam (with $C_1 = 1.13$) is obtained from Equation (2.13) of this publication:

$$M_{\mathrm{cr}} = C_1 \frac{\pi}{kL} \sqrt{EI_z GI_{\mathrm{T}}} \sqrt{1 + \frac{\pi^2 EI_{\mathrm{w}}}{(kL)^2 GI_{\mathrm{T}}}}$$

$$= 1.13 \frac{\pi}{5000} \sqrt{210000 \times 280 \times 10^4 \times 81000 \times 8.79 \times 10^4} \times$$

$$\sqrt{1 + \frac{\pi^2 \times 210000 \times 0.0812 \times 10^{12}}{9000^2 \times 81000 \times 8.79 \times 10^4}} \times 10^{-6}$$

$$= 64.1 \text{ kNm}$$

Calculate r_{n}:

$$d_{\mathrm{c1}}/h \quad = 50/349 = 0.14 \le 0.15$$

Therefore, $\alpha = \dfrac{h}{2(h - d_{\mathrm{c1}})} + \dfrac{2c}{h} = \dfrac{349}{2(349 - 50)} + \dfrac{2(100)}{349} = 1.15$

Hence,

$$r_{\mathrm{n}} = 1.0 - \frac{\alpha}{C_1 L} \sqrt{\frac{EI_{\mathrm{w}}}{GI_{\mathrm{T}}}} = 1.0 - \frac{1.15}{1.13 \times 5000} \sqrt{\frac{210000 \times 0.0812 \times 10^{12}}{81000 \times 8.79 \times 10^4}} = 0.68$$

Section 3.4.2 The reduced elastic critical moment allowing for the notch $M_{\mathrm{cr,n}}$ is given by:

$$M_{\mathrm{cr,n}} = r_{\mathrm{n}} M_{\mathrm{cr}} = 0.68 \times 64.1 = 43.8 \text{ kNm}$$

The non-dimensional slenderness is:

$$\bar{\lambda}_{\mathrm{LT}} = \sqrt{\frac{W_{\mathrm{pl,y}} f_{\mathrm{y}}}{M_{\mathrm{cr,n}}}} = \sqrt{\frac{543 \times 10^3 \times 275}{43.8 \times 10^6}} = 1.85$$

$$h/b \quad = 349.0/125.4 = 2.78$$

3-1-1/NA.2.17 For the case of rolled and equivalent welded sections, for I sections with $2 < h/b \le 3.1$, use buckling curve ‘c’ ($\alpha = 0.49$). For rolled sections, $\beta = 0.75$ and $\bar{\lambda}_{\mathrm{LT,0}} = 0.4$.

Hence,

3-1-1/6.3.2.3

$$\Phi_{\mathrm{LT}} = 0.5[1+\alpha_{\mathrm{LT}}(\bar{\lambda}_{\mathrm{LT}}-0.4)+0.75\bar{\lambda}_{\mathrm{LT}}^2]$$

$$= 0.5[1+0.49(1.85-0.4)+0.75\times1.85^2] = 2.13$$

$$\chi_{\mathrm{LT}} = \frac{1}{\Phi_{\mathrm{LT}}+\sqrt{\Phi_{\mathrm{LT}}^2-0.75\bar{\lambda}_{\mathrm{LT}}^2}} = \frac{1}{2.13+\sqrt{2.13^2-0.75\times1.85^2}} = 0.28$$

The design buckling resistance moment is therefore given by:

3-1-1/6.3.2.1

$$M_{\mathrm{b,Rd}} = \frac{\chi_{\mathrm{LT}}W_{\mathrm{pl,y}}f_{\mathrm{y}}}{\gamma_{\mathrm{M1}}} = \frac{0.28\times543\times10^3\times275}{1.0}\times10^{-6} = 42.1\,\mathrm{kNm}$$

3 Restraint forces acting on a bracing system

3.1 Introduction

A portal-frame building, 9 m wide, 5 m high and 15 m long, is braced in one of the end bays by means of cross-bracing, as shown in Figure 3.1. The bracing system provides lateral restraint to six roof beams. The roof beams are all 356 × 171 × 51 UKB sections and the bracing members are CHS 114 × 5.0 sections.

The design value of the total wind force acting on the gable ends of the building is 0.8 kN/m^2. The maximum design moment in the beams coexisting with this wind load is 260 kNm. Determine the forces acting on the bracing system.

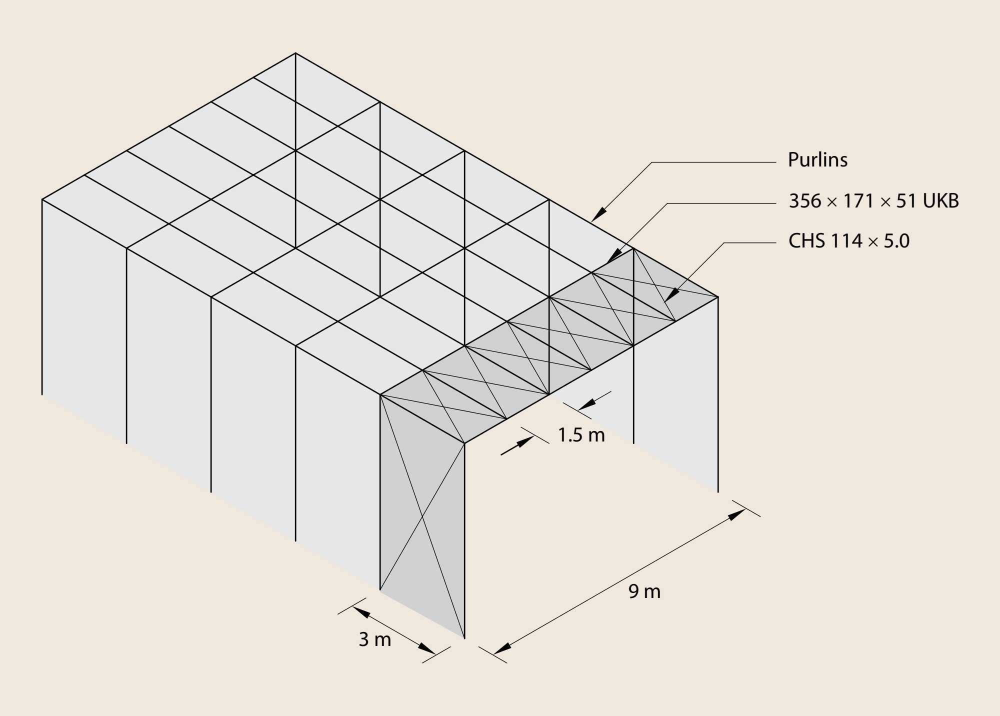

Figure 3.1 Structural arrangement

3.2 Imperfection for analysis of bracing system

3-1-1/5.3.3(1) In order to calculate the forces in the bracing, the imperfection in the members to be restrained must first be determined. The equivalent geometric imperfection of members to be restrained is given by $e_0 = \alpha_m L / 500$.

For this example, there are six beams to be restrained (i.e. $m = 6$) and the reduction factor on the imperfection for multiple beams is given by:

3-1-1/5.3.3(1)

$$\alpha_m = \sqrt{0.5\left(1+\frac{1}{m}\right)} = \sqrt{0.5\left(1+\frac{1}{6}\right)} = 0.764$$

The corresponding value of the imperfection is therefore:

$$e_0 = \alpha_m L / 500 = 0.764 \times 9000 / 500 = 13.7 \text{ mm}$$

3.3 Force in compression flange of beams

3-1-1/5.3.3(3) Clause 5.3.3(3) states that the force N_{Ed} in the compression flange of the beams to be stabilized may be taken as M_{Ed}/h, where M_{Ed} is the maximum design bending moment in the beam and *h* is the overall beam height. Assuming that all the beams are equally loaded, the total force in all 6 compression flanges is:

$$\sum N_{\mathrm{Ed}} = mM_{\mathrm{Ed}} / h = 6 \times 260 / 0.358 = 4360\,\mathrm{kN}$$

3.4 Equivalent stabilizing force

The equivalent stabilizing force q_{d}, initially assuming that the deflection of the bracing system $\delta_{\mathrm{q}} = 0$ can now be determined:

3-1-1/5.3.3(2)

$$q_{\mathrm{d}} = \sum N_{\mathrm{Ed}} 8 \frac{e_0 + \delta_{\mathrm{q}}}{L^2} = 8 \times 4360 \frac{(13.7 + 0)}{9^2} \times 10^{-3} = 5.92\ \mathrm{kN/m}$$

This force is applied to the bracing system, together with other external loads in the load combination, which in this example is the design wind force on the gables ($= 0.8 \times 5/2 = 2.0\ \mathrm{kN/m}$). The total design force is thus 7.92 kN/m.

Using analysis software, it is found that the maximum deflection of the bracing system due to this design force is 0.406 mm.

This value of δ_{q} is used to determine a revised value of q_{d}, which now allows explicitly for the flexibility of the bracing system:

$$q_{\mathrm{d}} = \sum N_{\mathrm{Ed}} 8 \frac{e_0 + \delta_{\mathrm{q}}}{L^2} = 8 \times 4360 \frac{(13.7 + 0.406)}{9^2} \times 10^{-3} = 6.07\ \mathrm{kN/m}$$

The deflection of the bracing system due to this enhanced design force is $\delta_{\mathrm{q}} = 0.422\ \mathrm{mm}$, which is sufficiently close to the original value of 0.406 mm to preclude the need for further iteration.

The bracing system should therefore be designed for a total design force of $6.07 + 2.0 = 8.07\ \mathrm{kN/m}$ to both stabilize the beams and resist the external loading.

Note that the total force applied to the bracing system in order to stabilize the beams is $6.07 \times 9.0 = 56.4\ \mathrm{kN}$, while the sum of the forces in the beam compression flanges ΣN_{Ed} is 4360 kN. The bracing system therefore provides a restraint force equivalent to $(54.6/4360) \times 100 = 1.25\%$ of ΣN_{Ed}.

4 Restraint from steel decking

4.1 Introduction

A composite floor system comprises trapezoidal steel decking and in-situ cast concrete. The decking is 6 m long supported on 8 m long 254 × 102 × 22 UKB beams at 3 m centres, as shown in Figure 4.1. The decking, which spans perpendicular to the beams and is attached at every rib, has a profile depth of 100 mm and thickness of 1.0 mm. Investigate whether, during the construction phase (prior to concrete hardening), the steel decking offers sufficient lateral restraint to the beams such that the beams may be considered as fully restrained against lateral torsional buckling.

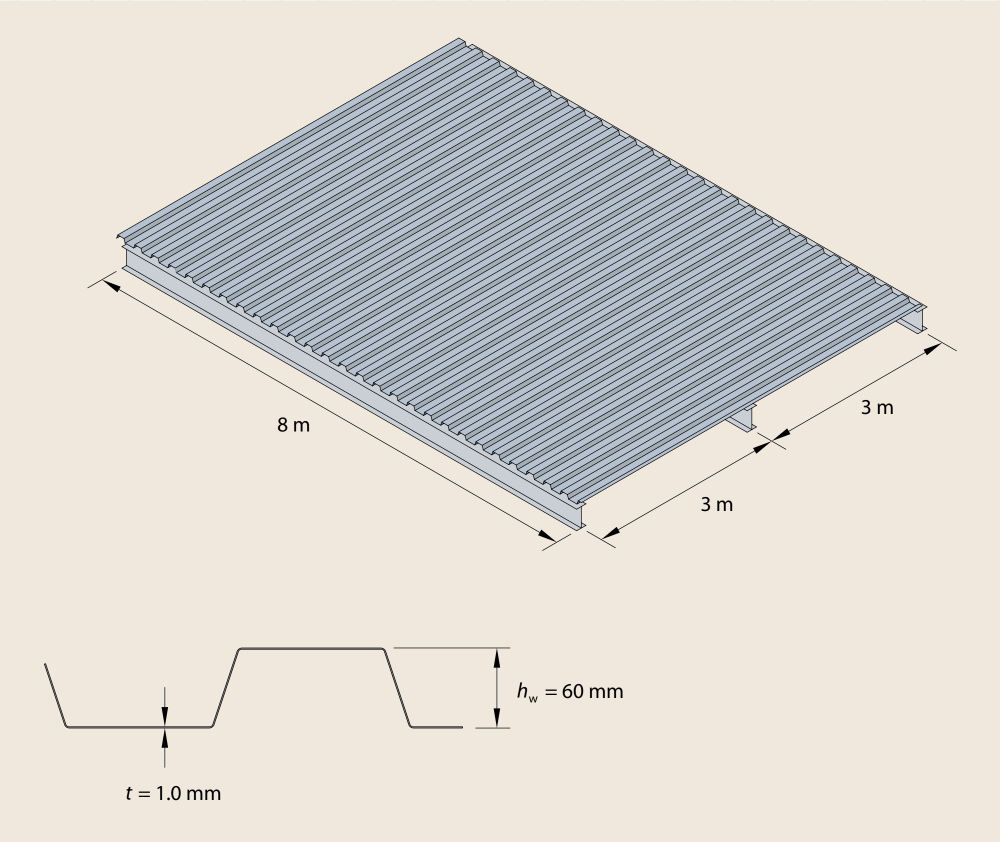

Figure 4.1 Structural arrangement and steel decking

4.2 Material properties

3-1-1/3.2.6 $E = 210000\ \mathrm{N/mm^2}$

$G = 81000\ \mathrm{N/mm^2}$

4.3 Sheeting profile properties

Overall length of sheeting perpendicular to the beams $b_r = 6000\ \mathrm{mm}$

Beam spacing	s	$= 3000\ \mathrm{mm}$
Decking thickness	t	$= 1.0\ \mathrm{mm}$
Decking profile height	h_w	$= 60\ \mathrm{mm}$

4.4 Beam properties

SCI P363 Section properties for the 254 × 102 × 22 UKB are:

h $= 254$ mm

I_z $= 119$ cm^4

I_T $= 4.15$ cm^4

I_w $= 0.0182$ dm^6

4.5 Required sheeting stiffness

For fully effective lateral restraint to the steel beam, the required sheeting stiffness S_{reqd} is given by:

3-1-1/BB.2.1(1)

$$S_{\text{reqd}} = \frac{70}{h^2}\left(EI_w\frac{\pi^2}{L^2} + GI_T + 0.25h^2EI_z\frac{\pi^2}{L^2}\right)$$

$$= \frac{70}{254^2}\left((210000\times0.0182\times10^{12}\times\frac{\pi^2}{8000^2}) + (81000\times4.15\times10^4) + (0.25\times254^2\times210000\times119\times10^4\times\frac{\pi^2}{8000^2})\right)$$

$= 4960000$ N per unit length

$= 4960$ kN per unit length

4.6 Available sheeting stiffness

The shear stiffness of trapezoidal sheeting is given by:

BS EN 1993-1-3 10.1.1(10)

$$S = 1000\sqrt{t^3}\,(50 + 10\sqrt[3]{b_r})\,\frac{s}{h_w}$$

$$= 1000\times\sqrt{1.0^3}\,(50 + 10\times\sqrt[3]{6000})\times\frac{3000}{60}\times10^{-3}$$

$= 11600$ kN per unit length

Since $S > S_{\text{reqd}}$, the floor beams may be considered to be laterally restrained by the sheeting during the construction phase and therefore the beam may be considered to be laterally restrained.

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The stability of beams and columns and the determination of their buckling resistance is an integral part of the design of steel buildings, which requires an understanding of the restraint conditions and buckling behaviour of the members. This publication provides immediate practical guidance for both the common and the non-standard restraint conditions commonly found in construction. The explanation of the underlying physics and the numerical worked examples form a comprehensive resource for all designers completing member buckling checks.

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