

# Design of Haunched Composite Beams in Buildings

R M Lawson BSc(Eng), PhD, ACGI, CEng, MICE, MStructE  
and J W Rackham BSc(Eng), MSc, DIC, CEng, MICE

ISBN: 1 870004 36 1  
© The Steel Construction Institute 1989

The Steel Construction Institute  
Silwood Park  
Ascot  
Berkshire SL5 7QN  
Telephone 0990 23345  
Fax 0990 22944  
Telex 846843

# FOREWORD

This publication was prepared by Dr R M Lawson and Mr J W Rackham. It is one of a series of publications on the design of long span composite beams in buildings. Others in the series are:

- *Design for openings in the webs of composite beams*
- *Design of fabricated composite beams in buildings*
- *Parallel beam approach – a design guide*

The design method presented in this publication is intended to be consistent with *BS 5950:Part 1* and *:Part 3.1* (in draft at the time of publication). The notation and methodology follows these standards, where appropriate. The 1985 draft of *Eurocode 4* was also used to provide additional guidance and, to encourage familiarity, some key words from *EC4* have been incorporated.

The following SCI members and staff commented on the publication:

B W J Boys	British Steel General Steels
B A Brown	Conder (Group Services) Ltd
I C Calder	Scott Wilson Kirkpatrick and Partners
Prof R P Johnson	University of Warwick
D L Mullett	The Steel Construction Institute
Dr G W Owens	The Steel Construction Institute
P J Wickens	Mott Hay and Anderson.

The research and design studies leading to this publication were partially funded by British Steel General Steels.

# CONTENTS

	Page
<b>SUMMARY</b>	v
<b>TERMINOLOGY</b>	v
<b>NOTATION</b>	vi
<b>1. INTRODUCTION</b>	1
<b>2. STRUCTURAL OPTIONS FOR LONG SPAN BEAMS</b>	2
<b>3. REVIEW OF DESIGN OF COMPOSITE BEAMS</b>	6
3.1 Simple composite beams	6
3.2 Continuous composite beams	6
3.3 Haunched composite beams	7
<b>4. MOMENTS AND FORCES IN HAUNCHED COMPOSITE BEAMS</b>	8
4.1 Global moments and forces in no-sway frames – elastic analysis	8
4.2 Plastic hinge analysis	10
4.3 Global moments and forces in sway frames	11
<b>5. ANALYSIS OF COMPOSITE SECTIONS</b>	12
5.1 Section classification	12
5.2 Analysis of composite section – positive (sagging) moment	12
5.3 Partial shear connection	14
5.4 Analysis of the composite section – negative (hogging) moment	15
5.5 Combined moment and shear	16
5.6 Transverse reinforcement	16
<b>6. ANALYSIS OF HAUNCHED SECTION</b>	17
<b>7. LATERAL STABILITY OF HAUNCHED COMPOSITE BEAMS</b>	19
7.1 Lateral stability of non-composite beams	19
7.2 Lateral stability of composite beams	20
7.3 Restraint forces	22
<b>8. SERVICEABILITY BEHAVIOUR OF COMPOSITE BEAMS</b>	23
8.1 Elastic section properties	23
8.2 Stresses in continuous and haunched composite beams	24
8.3 Deflection of continuous and haunched composite beams	24
8.4 Dynamic sensitivity	26
<b>9. APPLICATION OF HAUNCHED BEAMS</b>	27
<b>10. DESIGN OF HAUNCHED CONNECTIONS</b>	29
10.1 General principles	29
10.2 Interim design procedure for end plate connections	30
<b>11. DESIGN OF COLUMNS</b>	32
<b>12. SCHEME DESIGN OF HAUNCHED COMPOSITE BEAMS</b>	33

<b>13. DESIGN PROCEDURES</b>	<b>34</b>
<b>14. PRACTICAL FEATURES AND DETAILS</b>	<b>36</b>
14.1 Steel grade	36
14.2 Welding	36
14.3 Bolt spacing and grade	36
14.4 Tolerances	36
14.5 Connection of secondary elements	37
14.6 Precambering	37
<b>REFERENCES</b>	<b>38</b>
<b>Appendix A DESIGN EXAMPLE OF HAUNCHED COMPOSITE BEAM</b>	<b>39</b>

## SUMMARY

This publication presents a method of design for haunched composite beams as used in buildings. Moment continuity is developed between beams and columns by 'haunches', i.e. local deepening of the beam section. The publication describes two approaches to determining the global moments in the structure: by elastic design or by plastic hinge analysis. The moment resistance of the composite section is based on plastic section analysis in both cases.

Checks are made on the lateral stability of the beams both in the construction and in-service conditions. Serviceability calculations are made for deflection, stresses and vibration response. The publication also includes a detailed procedure for design, including that of the connections. Initial sizing of the members is also included in the Scheme Design. A fully worked design example is presented.

The publication is intended to be consistent with *BS 5950:Part 1* and *:Part 3.1* (which will be published in late-1989) and broadly with *Eurocode 4* (1985 draft).

## TERMINOLOGY

The following terms are used to encourage familiarity with *Eurocode 4*:

<i>Class 1 section</i>	section that can be used in plastic hinge analysis
<i>Global analysis</i>	determination of moments in frame
<i>Moment resistance</i>	bending capacity of member
<i>Negative moment</i>	hogging moment in beam
<i>Plastic hinge analysis</i>	development of plastic failure mechanism of continuous beam
<i>Plastic section analysis</i>	development of plastic stress blocks in section
<i>Positive moment</i>	sagging moment in beam

# NOTATION

$A$	cross-sectional area of steel beam
$B$	flange breadth of steel beam
$B_e$	effective breadth of concrete flange
$d$	depth of web (between flanges)
$D$	overall depth of beam
$D_p$	depth of deck profile
$D_r$	distance of reinforcement from top of steel flange
$D_s$	depth of concrete slab
$f_{cu}$	cube strength of concrete
$F_v$	shear force applied to steel section
$h$	length of column between floors
$I$	second moment of area of steel section
$I_{bc}$	second moment of area of composite section
$I_c$	second moment of area of column
$K$	degree of shear connection
$L$	span of haunched beam
$L_e$	distance between the tips of the haunches
$M_c$	plastic moment resistance of composite section including the effects of partial shear connection
$M_{hc}$	elastic moment resistance of deepest section of haunch
$M_{nc}$	negative (hogging) moment resistance of composite section
$M_{pc}$	positive (sagging) moment resistance of composite section
$M_s$	plastic moment resistance of steel section
$n$	slenderness correction factor
$p_y$	design strength of steel
$P_v$	shear resistance of web
$R_c$	compressive resistance of effective breadth of concrete flange
$R_q$	longitudinal resistance of shear connectors in the zone of positive or, alternatively, negative moment
$R_s$	tensile resistance of steel section
$R_v$	tensile resistance of steel web (depth $d$ )
$R_w$	tensile resistance of steel web (depth $D$ )
$t$	web thickness
$T$	flange thickness
$v_t$	slenderness factor (including torsional and distortion effects)
$w_u$	ultimate (factored) uniformly distributed load on beam
$x$	torsional index
$x_e$	modular ratio between steel and concrete
$y_e$	depth of elastic neutral axis below top of slab
$\phi_c$	parameter $I_c L / (I_{bc} h)$
$\lambda$	slenderness of beam
$\lambda_{LT}$	effective slenderness of beam under lateral torsional buckling

# 1. INTRODUCTION

Composite buildings comprising steel frames and concrete floors combine greater structural economy with a faster speed of construction than non-composite or concrete structures. The use of steel decking is an integral part of the structural system as it supports the load developed before and during concreting, and later acts compositely with in-situ concrete to form a composite slab. Shear connectors develop composite action between the steel beams and the concrete. Various publications describe this method of construction<sup>(1, 2, 3)</sup>.

Composite beams are usually of simple construction, i.e. no account is taken of the moment continuity provided by the beam-to-column or beam-to-beam connections. This is mainly because of ease of design and construction, but also partly because adequate structural performance can readily be achieved by developing composite action alone. This is certainly true for beam spans of 6 m to 10 m, which form the bulk of those currently specified.

However, there is now a strong demand for longer column-free spans in buildings, either for open-planning or to offer greater flexibility in office layout. For longer spans the selection of the appropriate structural form is more difficult. Conventional simple construction may still be used, but often the size of the beams is such that the floor zone is excessively deep. This problem is compounded by the need to incorporate a high degree of servicing in modern buildings, most of which is located beneath the structural floor zone.

Various design solutions are feasible, but there are two basic options: either the structure and services are integrated within the same horizontal zone or the structural zone is minimized so that the services are passed beneath. These solutions are described in simple terms in the following section.

The economics of the design of modern buildings is such that the costs of the frame rarely exceed 15% of the total cost of the completed building. This means that the structural cost itself is not necessarily indicative of overall economy. Many of the solutions adopted represent a nominal increase in material and fabrication cost but permit greater flexibility in building use and servicing.

One of the potential solutions for beam spans in the region of 15 to 20 m is the haunched beam. This form of construction is more readily associated with portal frames, but it is appropriate to draw on some of its advantages for wider use in buildings. By developing continuity, beam moments and deflections are reduced at the expense of increased column moments. Nevertheless, this can lead to overall economy by enabling the use of shallower and lighter beams.

This publication describes the features of haunched composite beams and puts forward a design method consistent with *BS 5950:Part 1*<sup>(4)</sup> and *:Part 3.1*<sup>(5)</sup> (currently in draft). Haunched beams can also be used to advantage where the frame is to be designed for lateral load resistance (i.e. as a sway frame).

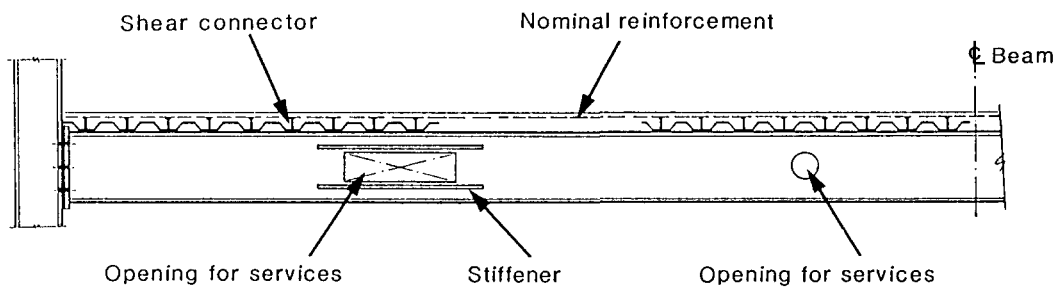
## 2. STRUCTURAL OPTIONS FOR LONG SPAN BEAMS

Composite slabs are usually designed to span 3 to 4 m between support beams and their depth is typically 120 to 150 mm. This dictates the economic layout of the structural grid. The long span beams under consideration may be loaded directly by the composite slab or loaded by secondary beams which support the slab.

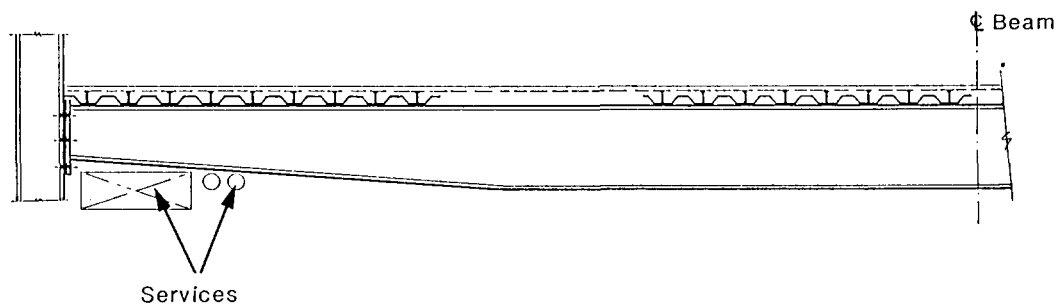
The various structural options for achieving the twin aims of long spans and ready incorporation of services within normal floor zones include:

- *Beams with web openings*

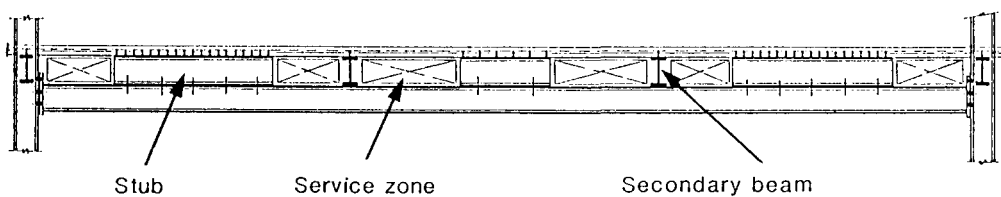
In this method of construction, the depth of the steel beam is selected so that sufficiently large, usually rectangular-shaped openings can be cut into the web (see Figure 1(a)). For general guidance, it is suggested that the openings should form no more than 70% of the depth of the web, where horizontal stiffeners are welded above and below the opening. Typically, the length of the opening should be no more than 2 times the beam depth. The best location of the openings is in the low shear zone of the beams. A step by step method of design is presented in the SCI/CIRIA publication *Design for openings in the webs of composite beams*<sup>(6)</sup>.



(a) BEAM WITH WEB OPENINGS



(b) TAPERED BEAM



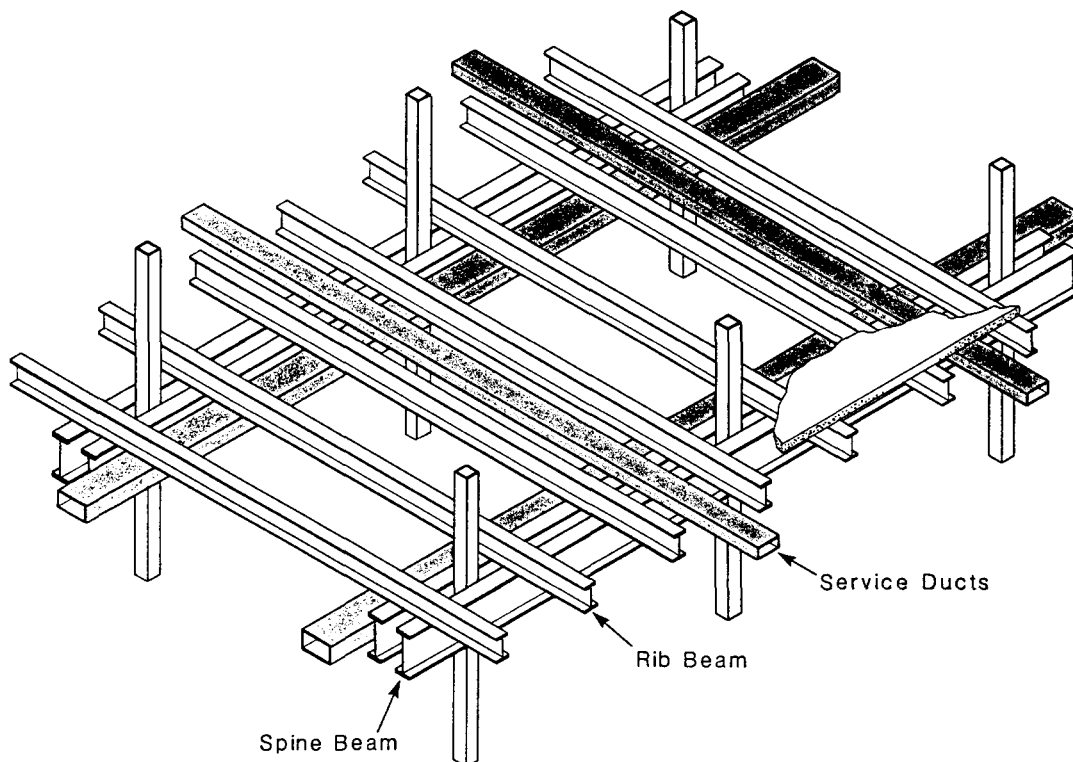
(c) STUB GIRDER

**Figure 1** Different methods of incorporating services within the structural depth



A modified form of construction is the notched beam where the lower section of web and flange of the section is cut away over a short distance from the support. This method is not usually practical unless the cut web is stiffened.

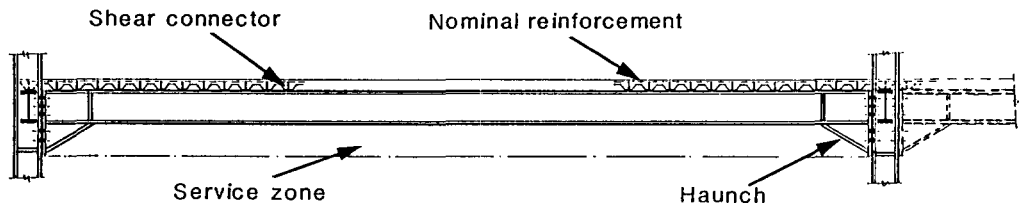
- *Castellated beams*  
Castellated beams can be used effectively for lightly serviced buildings or for aesthetic reasons where the structure is exposed. Composite action does not significantly increase the strength of the beams but increases their stiffness. Castellated beams have limited shear capacity and are best used as long span secondary beams or where loads are relatively low. The design of castellated beams is covered by an SCI publication<sup>(7)</sup> which gives design tables for standard non-composite castellated sections.
- *Fabricated beams with tapered webs*  
The tapered web beam is designed to provide the required moment and shear capacity at all points along the beam, and the voids created adjacent to the columns can be used for modestly sized service runs. Typically, the tapered beam is most economic for spans of 13 to 20 m. The plate sizes can be selected for optimum structural performance, and the plates welded in an automatic single-sided submerged arc process. Thicker webs are welded by double-sided fillet welds. Web stiffeners are often required at the change of section when taper angles exceed approximately 6°. A typical tapered beam is shown in Figure 1(b).
- *Trusses*  
Trusses are frequently used in multi-storey buildings in North America and are best suited for very long spans, where the truss is designed to occupy the full depth of the floor zone. The cost of fabrication can be high in relation to the material cost but trusses can be cost-effective and have been used in a number of major projects. Little benefit is gained from composite action apart from improving the stiffness of the truss. The modified Warren truss is the most common form as it offers the maximum zone for services between bracing members.
- *Stub girders*  
Architectural demand for square column grids with spacings of 10 to 12 m led to the development of stub girder construction in North America. The stub girder comprises a bottom chord which acts in tension and a series of short beam sections (or stubs) which connect the bottom chord to the concrete slab. Secondary beams span across the bottom chord and can be designed as continuous members. Voids are created adjacent to the stubs for services. This is illustrated in Figure 1(c).  
The major disadvantage of the conventional stub girder is that it requires temporary propping until the concrete has gained adequate strength for composite action. However, it is possible to introduce a light steel top chord, such as a T-section, which acts on compression to develop the required bending strength of the girder during construction.
- *Parallel beam grillage systems*  
This system is different from the others previously described in that continuity can be developed in both the secondary and primary beams. The secondary beams are designed to act compositely with the concrete slab, and are made continuous by passing over the primary beams. The primary beams are arranged in pairs and pass on either side of the columns, to which they are attached by shear-resisting brackets. These primary beams are non-composite. The method of construction is illustrated in Figure 2. Parallel beam systems are ideally suited to accommodating large service ducts in orthogonal directions.
- *Haunched beams*  
Haunched beams are designed by forming a rigid moment connection between the beams and columns. The depth of the haunch is selected primarily to provide an economic method of transferring moment into the column. The length of the haunch is selected to reduce the depth of the beam to a practical minimum. The extra service zone created beneath the beam between the haunches offers flexibility in service layout.



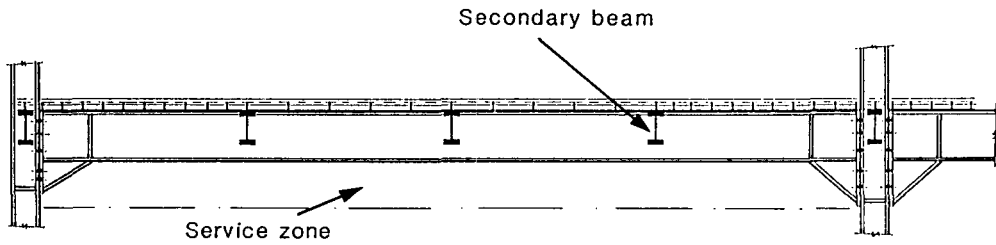
**Figure 2** *Parallel beam grillage system showing service zones*

At edge columns, it would not be normal practice to develop additional continuity through the slab reinforcement, but this is an option at internal columns. This form of construction can be used for sway frames, i.e. where vertical bracing or concrete shear walls or cores are not provided. It is practical for buildings up to 5 storeys in height but is generally uneconomic in comparison to braced construction in taller buildings.

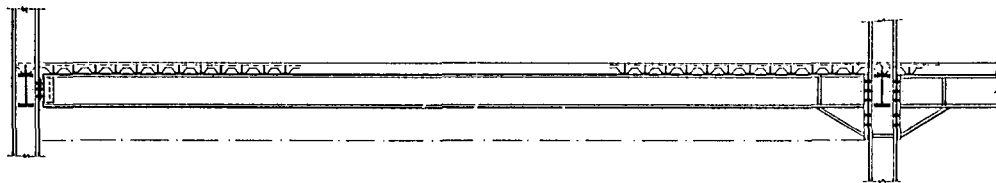
Examples of different forms of haunched composite beams are shown in Figure 3.



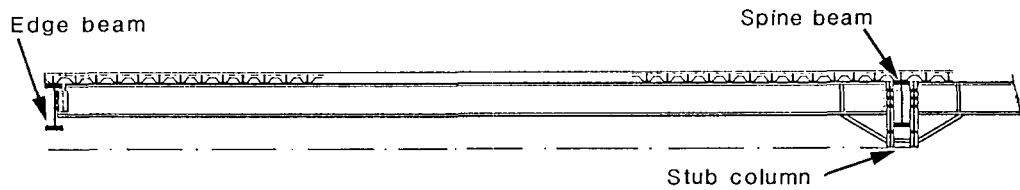
(a) HAUNCHED BEAM SUPPORTING COMPOSITE SLAB - SINGLE OR MULTI-BAY FRAME



(b) HAUNCHED BEAM SUPPORTING SECONDARY BEAMS - SINGLE OR MULTI-BAY FRAME



(c) SINGLE SIDED HAUNCHED BEAM - MULTI-BAY FRAME



(d) HAUNCHED BEAM CONNECTED TO SPINE BEAM

**Figure 3** *Different configuration of haunched composite beams*

## 3. REVIEW OF DESIGN OF COMPOSITE BEAMS

### 3.1 Simple composite beams

The design of composite beams is presented in *BS 5950:Part 3.1*<sup>(5)</sup> and it is assumed that the designer is familiar with the general approach to the analysis of composite sections. In principle, simple composite beams are designed to meet strength and serviceability criteria. Plastic analysis of the section is usually employed for strength calculations, and elastic analysis for serviceability calculations. The effective breadth of concrete considered to act with each steel beam is taken as 25% of the beam span but not exceeding the beam spacing (or 80% of the beam spacing when the slab and beam span in the same direction<sup>(5)</sup>). The same effective breadth is used in strength and serviceability calculations.

The main advantages of composite relative to non-composite steel beams are:

- savings in steel weight of 30% to 50%
- greater stiffness, leading to shallower beams for the same span.

It is normally found that strength and serviceability design limits are just satisfied when the ratio of beam span to overall depth (including the concrete or composite slab) is between 18 and 22. This usually represents the optimum design of simple composite beams.

Full shear connection exists when sufficient shear connectors are provided to develop the full plastic resistance of the section. Design strengths of shear connectors are given in *BS 5950:Part 3.1* and *BS 5400:Part 5*<sup>(8)</sup>.

Design for full shear connection results in the lightest beam. Where fewer shear connectors are provided (known as partial shear connection) the beam is heavier. However, the overall design may be more practical and economic by arranging the shear connectors in a standard pattern, e.g. one per trough of the deck profile, and designing for partial shear connection.

### 3.2 Continuous composite beams

Moments and forces in continuous beams or frames can be determined from elastic global analysis or, alternatively, from plastic hinge analysis where the section is 'plastic' according to *BS 5950:Part 3.1*. In elastic global analysis the concrete is usually assumed to be uncracked when evaluating the elastic section properties. Redistribution of moment from the positive (sagging) moment region to the negative (hogging) moment region is permitted, depending on the section classification.

The positive moment resistance of a continuous composite beam is evaluated as for a simple composite beam. The effective breadth of the slab and the degree of shear connection provided are based on the zone of the beam subject to positive moment (conservatively, the effective breadth is taken as  $0.7 \times \text{span}/4$  subject to the same limitation as for simple spans).

The negative moment resistance of the composite beam is evaluated from the moment resistance of the steel section and properly anchored reinforcement in the slab. Welded mesh is discounted in this calculation. The effective breadth of the slab is based on the zone of the beam subject to negative moment (conservatively, the effective breadth is taken as  $0.5 \times \text{span}/4$  for internal spans). The effect of the tensile reinforcement is to create a deeper zone of the web subject to compression. This tensile force is developed by an appropriate number of shear connectors in the negative moment region.

It is normally found that strength and serviceability design limits are just satisfied when the ratios of beam span to overall depth of continuous composite beams are between 22 and 25 for end spans and 25 and 28 for internal spans.

Special consideration should be given to the lateral stability of continuous composite beams in the negative moment region. Pattern loading arising during the concreting operation may influence the design of the plain steel section.

The approach used to calculate deflections of continuous composite beams assumes that the behaviour is elastic unless yielding takes place in the beam at the serviceability limit state. This occurs when the redistribution of support moment exceeds about 30% at the ultimate limit state in either elastic or plastic analysis. For greater redistributions, consideration is to be given to initial cycles of loading, leading to local 'plastic' rotation and increased deflections<sup>(5, 9)</sup>.

### 3.3 Haunched composite beams

Haunched composite beams are designed in a similar manner to continuous beams of uniform section. The critical section for design is in the beam at the tip of the haunch, as the depth of the haunch is selected principally to develop the required moment in the beam-to-column connection. The length of the haunch is selected to achieve an efficient design of the beam and would typically be 5 to 7% of the length of the beam. Greater haunch lengths (7 to 15%) may be required in sway frames to compensate for the greater length of the beam subject to negative (hogging) moment.

Haunched composite beams can be used in cases where the beams frame directly into the major axis of columns, and where the size of the columns is such that substantial moment can be transferred from the beam to the column. This means that heavier columns and more complex connections will be required in comparison with simply supported construction, but considerable economy is gained in the sizing of the beams.

In Scheme Design, it would be normal to neglect the continuity provided by the slab reinforcement, although this can be utilised in final design to gain further economy in the design of the haunch and its connection at internal beam-to-column junctions. The plastic moment resistance of the beam at the tip of the haunch can be developed in compact or plastic sections but it is normal to design the haunch elastically. In practice, the connection capacity would rarely exceed 80% of the elastic moment resistance of the haunch, and therefore little economy is gained in optimising the design of the haunch. Indeed, the haunch itself would normally be taken from a 'cutting' of the beam section, and the total depth of the haunch would be up to twice the beam depth.

The connection design is critical to the practicality of the system and this is covered in Section 10. Other considerations are the local transfer of force between the beam and the haunch which often necessitates the use of a web stiffener in the beam (Section 6). Lateral stability of the haunch and adjacent beam is covered in Section 7.

## 4. MOMENTS AND FORCES IN HAUNCHED COMPOSITE BEAMS

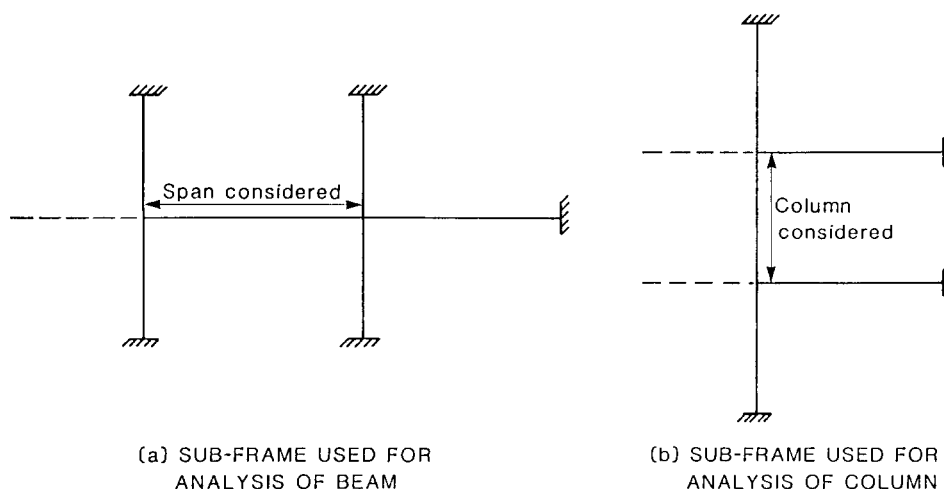
The design approach which follows is consistent with *BS 5950:Part 1* and *Part 3.1*. Reference is made to the design formulae in these Standards, defining the terms used, as appropriate. Where there is a lack of design information, a simplified design method has been developed.

### 4.1 Global moments and forces in no-sway frames – elastic analysis

Elastic analysis can be used for determining the moments and forces in all continuous beams and frames. Two approaches are valid: either gross (uncracked) section properties can be used, ignoring the haunch, or the properties of the haunch and other cracked section properties can be introduced in a generalised analysis.

In the analysis of continuous beams the designer is permitted to take a redistribution of moment from the negative (hogging) to the positive (sagging) moment regions of the beam (see Table 1, taken from *BS 5950:Part 3.1*). Part of this redistribution arises from cracking and loss of stiffness of the composite section and part from local yielding of the steel beam. Because allowance has already been made for cracking in the second approach, the permitted redistribution of moment is less. The classification of the steel section influences the degree of local yielding that is permitted.

For analysis of the beam members of no-sway frames under vertical loads, a sub-frame may be created by which the column ends remote from the beam under consideration are assumed to be fixed (or pinned at foundations) (see Figure 4 (a)). The sub-frame is then analysed elastically under various load combinations.



**Figure 4** Use of sub-frames in 'no-sway' frame analysis

The magnitude of the negative moment largely depends on the relative stiffness of the adjacent column and beam. If the beam stiffness is under-estimated, the negative beam moments and the column moments are over-estimated. The stiffness of the haunch largely compensates for any loss of stiffness of the beam due to concrete cracking. Ignoring both effects is generally conservative for braced frames as it is usually the consideration of the negative moment region that determines the sizing of the steel beam.

Taking the simple case of a single-bay haunched beam with columns above and below the beam being analysed, the negative moment at the beam ends is given by:



$$M_n = \left( \frac{4\phi_c}{4\phi_c + 1} \right) \text{FEM} \quad (1)$$

where FEM = the fixed-ended moment of the beam under the same loading conditions

and  $\phi_c$  = the parameter  $I_c L / (I_{bc} h)$

where  $I_c$  = the second moment of the area of the column

$h$  = the length of the column from floor to floor

$I_{bc}$  = the second moment of area of the composite beam (assumed to be uncracked)

$L$  = the length of the beam (including the haunch).

Other typical cases are given in Appendix A.

In elastic global analysis the length of the haunch does not significantly affect the applied bending moment. Therefore, the length of the haunch may be adjusted so that the moment resistance of the beam is compatible with the applied moment. There may be cases where the designer wishes to reduce the length of the haunch and in doing so the moment resistance of the beam falls below the applied moment. However, the positive moment resistance of the beam may greatly exceed the applied moment in mid-span.

Redistribution of moment may result in more economic design.

The maximum redistributions of moment in Table 1 therefore apply to the moment at the tip of the haunch (i.e. in the uniform section) as this is the zone potentially subject to greatest loss of stiffness due to steel yielding and concrete cracking. Equilibrium is maintained by increasing the positive moment by the magnitude of the redistributed moment.

**Table 1** Maximum redistribution of negative moment in haunched composite beams at ultimate limit state

Assumed section properties	Classification of beam section			
	Slender beam	Semi-compact beam	Compact beam	Plastic beam
Gross uncracked (haunch ignored)	10%	20%	30%	40%
Cracked under negative moment (haunch included)	0%	10%	20%	30%

*Redistribution applies to moment in beam at tip of haunch*

*Section classification as in BS 5950:Part 3.1<sup>(5)</sup>*

However, the effect of this moment redistribution is to reduce the moment in the haunch and consequently the moment transferred to the columns. If, in practice, the beam is stronger than assumed in design, the actual redistribution of moment resulting from loss of stiffness in this zone would be less, leading to higher moments in the haunch, connection or column. Potentially, these elements could undergo excessive deformation if they are not as strong as the beam.

Conservatively, the critical elements of the construction, i.e. the connections and columns, should be designed for the elastic moment prior to any redistribution of the beam moment. This approach is increasingly conservative for redistributions exceeding 20% of the beam moment and is incompatible with the approach adopted for plastic hinge analysis. It would be reasonable to permit use of Equation (3) below, in cases where plastic hinges are developed at the tips of the haunches in 'plastic' sections, assuming these points are laterally restrained. This is equivalent to an elastic redistribution of beam moment 10% less than that used in the ultimate load design.

A satisfactory 'strength' design is obtained when the moment resistance of the section and the connection exceeds the applied (or redistributed) moments at all points along the beam as illustrated in Figure 5.

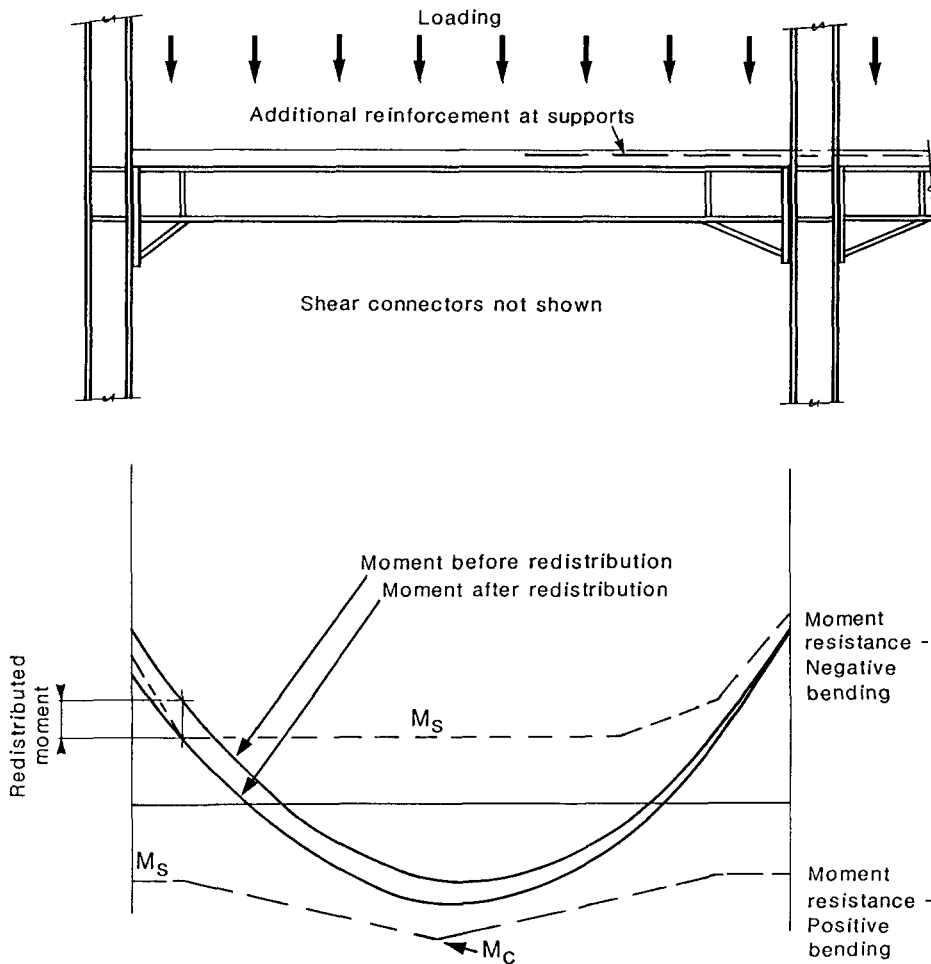


Figure 5 Redistribution of moment in haunched composite beams

## 4.2 Plastic hinge analysis

Plastic hinge analysis can only be used where the section is ‘plastic’ (or Class 1 to *Eurocode 4*), and where plastic hinge locations are laterally restrained. Plastic hinges are assumed to form in the beam at the ends of the haunches and at the point of maximum positive moment. This local beam mechanism should occur before failure of the connection or the column. It would be good practice to introduce an additional factor of safety into the design of the connection and to use this increased moment in designing the column to *BS 5950:Part 1*.

The collapse load of a uniformly loaded beam is defined by the plastic failure mechanism of the beam between the tips of the haunches, such that:

$$M_{pc} + M_{nc} \geq w_u \frac{L_c^2}{8} \tag{2}$$

where  $M_{pc}$  = the positive moment resistance of the composite beam (or  $M_c$  taking into account partial shear connection as in Section 5.3)

$M_{nc}$  = the negative moment resistance of the composite beam at the tip of the haunch (see Section 5.4)

$w_u$  = the factored design load on the beam

$L_c$  = the span of the beam between the ends of the haunches ( $L_c \approx 0.9L$ ).

For other loading arrangements, the plastic failure load of a beam may be determined from first principles.

Limitations on the use of this method are given in *BS 5950:Part 3.1*. In principle, the length of the end span should be between 75 and 115% of the length of the adjacent span to avoid development of other plastic mechanisms under pattern loading.



To ensure that failure of the haunch or connection does not occur, the haunch and its connection are designed to withstand the moments induced when the haunch tip moment is increased by 10% (i.e.  $1.1M_{nc}$ ). The connection moment in a uniformly loaded beam is determined from:

$$M_{hc} \geq M_{conn} \geq w_u \frac{(L^2 - L_c^2)}{8} + 1.1M_{nc} \quad (3)$$

where  $M_{hc}$  = the elastic moment resistance of the deepest section of the haunch

$M_{conn}$  = the moment resistance of the beam-to-column connection.

This formula is obtained assuming that the haunch is subject to plastic moment and a point reaction at one end and a local uniform load along its length. Similar expression may be derived for other forms of loading.

The factor of 1.1 is introduced so that any potential over-strength of the beam in a zone required to undergo 'plastic' rotation does not lead to excessive deformation of the connection or the column. It is appreciated that this is conservative with respect to the traditional design of haunched beams. Nevertheless, the behaviour of composite beams is such that the degree of the moment redistribution at failure could be up to 50% leading to considerable rotation at the plastic hinges. Further research may lead to a relaxation of this requirement.

The above approach is less conservative than that suggested for elastic design without redistribution of moment. Economic design is usually achieved by using shorter haunches than in elastic design, thereby limiting the moment transferred to the columns.

### 4.3 Global moments and forces in sway frames

The sub-frame approach can also be used in Scheme Design for elastic analysis of regular frames under lateral load. In this case the sub-frame consists of a typical storey-height frame but, to model the asymmetric bending action, pin joints are located at mid-height of the columns. Uncracked section properties are used. The shear force applied to the substitute frame is equal to the total wind force acting above and including the floor level under consideration, but not less than the notional forces in Clause 2.4.2.3 of *BS 5950:Part 1*.

Final design of the structure is to be carried out for an accurate distribution of moments as determined from a full analysis of the structure under lateral load (as noted in Clause 5.6.4.2 of *BS 5950:Part 1*). No redistribution of moment is permitted.

## 5. ANALYSIS OF COMPOSITE SECTIONS

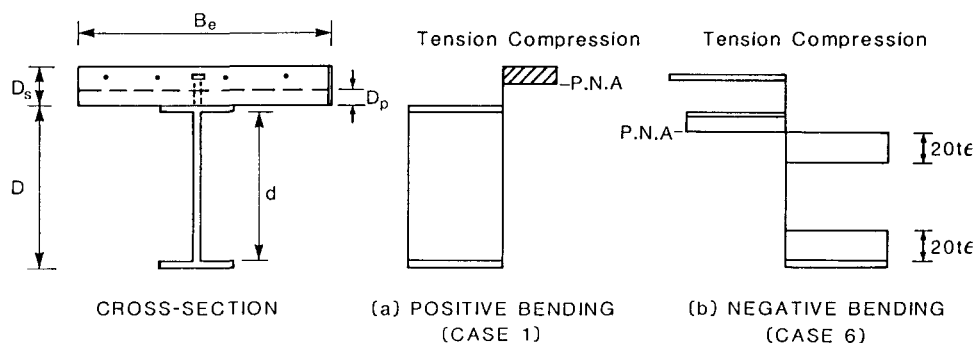
### 5.1 Section classification

The classification of the section depends on the proportions of the steel flange and web in compression. When subject to positive moment, the top flange of the beam is assumed to be fully restrained against local buckling provided it is connected to the concrete slab at sufficient points so that it can be designed as a composite section. The plastic neutral axis (P.N.A.) depth of the composite section is such that relatively little of the web (if any) is subject to compression. This means that for practical purposes composite beams comprising universal beam sections may be treated as 'plastic' or 'compact' when subject to positive moment, and plastic analysis of the section can be used.

When the composite beam is subject to negative moment the situation is different. Firstly, the lower flange is unrestrained and, secondly, more of the web is in compression if the slab reinforcement is included in evaluating the strength of the section.

The approach in *BS 5950:Part 3.1* differs from that in *Eurocode 4 (1985 draft)*<sup>(10)</sup> in that the section classification is expressed solely in terms of the proportions of the lower flange, as given in Table 7 of *BS 5950:Part 1*. Only in 'plastic' or 'compact' sections can the plastic moment resistance of the section be utilised. However, in order to develop 'rotational capacity' in plastic hinge analysis, the web in the zone of the 'hinges' should be 'plastic' or 'compact', and the lower flange should be plastic.

The treatment of the web in compression is unique to *BS 5950:Part 3.1*. When the zone of the web in compression exceeds  $40t\varepsilon$  where  $t$  is the web thickness and  $\varepsilon$  is  $(275/p_y)^{1/2}$ , a method is given for discounting the additional portion of the web when evaluating the plastic resistance of the effective section (see Figure 6).



**Figure 6** Examples of stress blocks used in the plastic analysis of composite sections

### 5.2 Analysis of composite section – positive (sagging) moment

The plastic moment resistance of the section is independent of the order of loading (i.e. propped or unpropped construction). The plastic neutral axis of the composite section is evaluated assuming stresses of  $p_y$  in the steel section and  $0.45 f_{cu}$  in the concrete. The tensile resistance of the steel section is therefore  $R_s = p_y A$  where  $A$  is the cross-sectional area of the beam. The compressive resistance of the concrete slab depends on the orientation of the decking. Where the decking crosses the beams the depth of concrete contributing to the compressive capacity is  $D_s - D_p$ . Clearly,  $D_p$  is zero in a solid slab. Where the decking runs parallel to the beams then the total cross-sectional area of the concrete may be used, although it is common practice to neglect the concrete in the deck troughs.

Taking the first case, the compressive resistance of the concrete is:

$$R_c = 0.45f_{cu} (D_s - D_p) B_c \tag{4}$$

where  $B_c$  = the effective breadth of the slab

$f_{cu}$  = the cube strength of concrete

$D_s$  = the slab depth

$D_p$  = the profile height.

Three cases of plastic neutral axis depth  $y_p$  (measured from the upper surface of the slab) exist. It is not necessary to calculate  $y_p$  explicitly if the following formulae for the plastic moment resistance of I section beams subject to positive moment are used.  $R_w$  is the axial resistance of the web and  $R_f$  is the axial resistance of one steel flange (the section is assumed to be symmetrical). The top flange is considered to be fully restrained by the concrete slab.

- *Case 1:*  $R_c > R_s$  (plastic neutral axis lies in concrete slab as in Figure 6(a))

$$M_{pc} = R_s \left[ \frac{D}{2} + D_s - \frac{R_s}{R_c} \left( \frac{D_s - D_p}{2} \right) \right] \tag{5}$$

- *Case 2:*  $R_s > R_c > R_w$  (plastic neutral axis lies in steel flange)

$$M_{pc} = R_s \frac{D}{2} + R_c \left( \frac{D_s + D_p}{2} \right) - \frac{(R_s - R_c)^2 T}{R_f 4} \tag{6}$$

Note: the last term in this expression is generally small ( $T$  is the flange thickness) and can usually be neglected.

- *Case 3:*  $R_c < R_w$  (plastic neutral axis lies in web)

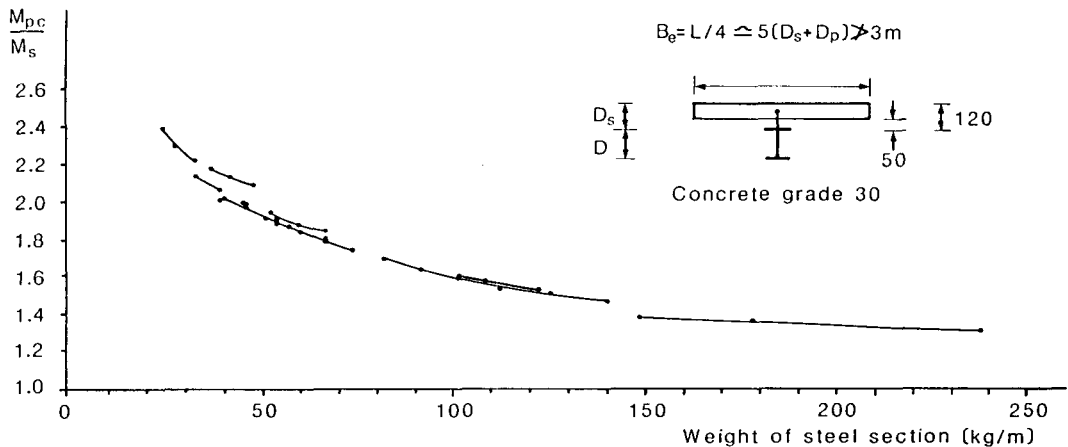
$$M_{pc} = M_s + R_c \left( \frac{D_s + D_p + D}{2} \right) - \frac{R_c^2 D}{R_w 4} \tag{7}$$

where  $M_s$  = the plastic resistance moment of the steel section alone

$D$  = the beam depth.

This formula assumes that the web is compact (i.e. not subject to the effects of local buckling). In this case, the depth of the web in compression should not exceed  $40t\varepsilon$  where  $t$  is the web thickness ( $\varepsilon$  is defined earlier). If the web is non-compact, a formula for determining the resistance of the section is given in *BS 5950:Part 3.1* Appendix B<sup>(1)</sup>.

The ratio of the plastic moment resistance of composite universal beams to non-composite beams varies with section size as shown in Figure 7. This is an approximate relationship because of the assumed properties of the slab. The concrete is taken as grade 30 lightweight concrete and the slab depth is 120 mm. The effective breadth of the slab is taken as 5 times the overall beam depth, but not exceeding 3 m.



**Figure 7** Ratio of plastic moment resistance of composite section to that of the steel section

## 5.3 Partial shear connection

In plastic section analysis of composite beams the longitudinal shear force to be transferred between the concrete and the steel for full shear connection is the lesser of  $R_c$  or  $R_s$ . The number of shear connectors placed along the beam between the points of zero and maximum positive moment should be sufficient to transfer this force. The strength of stud shear connectors is presented in Table 5 of *BS 5950:Part 3.1* and in *BS 5400:Part 5*<sup>(8)</sup>.

In cases where fewer shear connectors than the number required for full shear connection are provided it is not possible to develop  $M_{pc}$ . If the total shear resistance of the shear connectors between the points of zero and maximum positive moment is  $R_q$  (less than the smaller of  $R_s$  and  $R_c$ ) then the stress block method in the previous section may be modified as follows:

- *Case 4:*  $R_q > R_w$  (plastic neutral axis lies in flange)

$$M_c = R_s \frac{D}{2} + R_q \left[ D_s - \frac{R_q}{R_c} \left( \frac{D_s - D_p}{2} \right) \right] \quad (8)$$

- *Case 5:*  $R_q < R_w$  (plastic neutral axis lies in web)

$$M_c = M_s + R_q \left[ \frac{D}{2} + D_s - \frac{R_q}{R_c} \left( \frac{D_s - D_p}{2} \right) \right] - \frac{R_q^2 D}{R_w 4} \quad (9)$$

where  $M_c$  = the positive moment resistance including the effects of partial shear connection ( $< M_{pc}$ ).

The above formulae are obtained by replacing  $R_c$  by  $R_q$  and re-evaluating the neutral axis position. This stress-block method is similar to that used in the American method of plastic design<sup>(11)</sup>. It predicts a non-linear increase of moment resistance with degree of shear connection  $K$  defined as:

$$K = \frac{R_q}{R_s} \quad \text{for } R_s < R_c$$

or

$$K = \frac{R_q}{R_c} \quad \text{for } R_c < R_s$$

An alternative approach<sup>(1,2)</sup> which has proved attractive is to define the moment resistance in terms of a linear interaction with the degree of shear connection, such that:

$$M_c = M_s + K(M_{pc} - M_s) \quad (10)$$

The 'stress block' and 'linear interaction' methods are presented in Figure 8 for a typical beam. It can be seen that there is a significant benefit in the stress block method in the important range of  $K = 0.5$  to  $0.7$ .

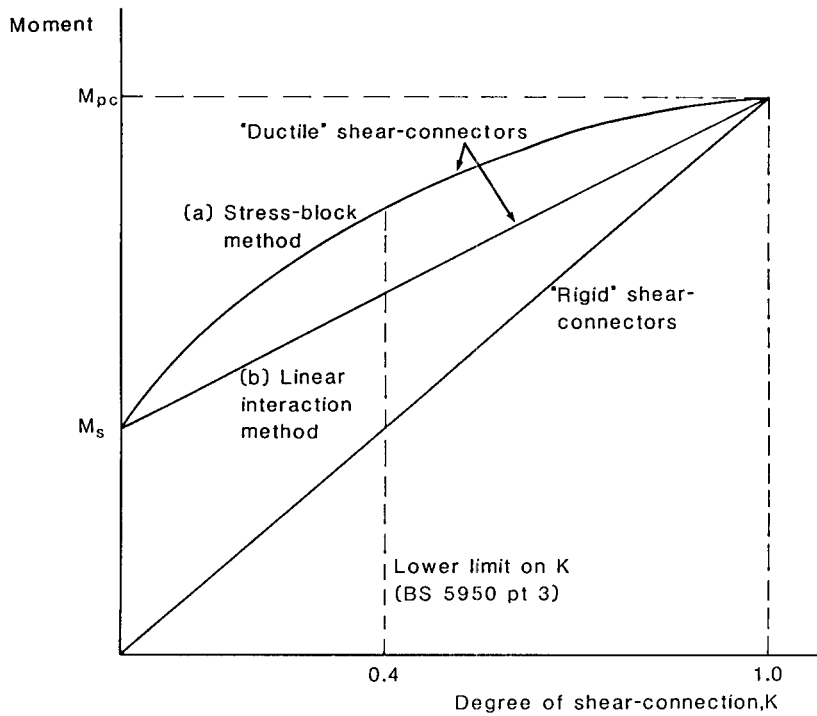
In using methods based on partial shear connection a lower limit for  $K$  of  $0.5$  is specified in *Eurocode 4* (draft)<sup>(10)</sup>. This is to overcome any adverse effects arising from the limited deformation capacity of the shear connectors.

In *BS 5950:Part 3* the minimum degree of shear connection to be developed increases with span ( $L$  in metres) such that:

$$K \geq \frac{L - 6}{10} \geq 0.4$$

This formula means that beams longer than 16 m span are to be designed for full shear connection, and beams of up to 10 m span may be designed for 40% shear connection. Partial shear connection is also not permitted for beams subject to heavy point loads applied close to the beam supports. It can be used for beams subject to point loads from secondary beams.

A further requirement is that the degree of shear connection should be adequate at all points along the beam length. For a beam subject to point loads, it follows that the shear connectors may be distributed in proportion to the area under the shear force diagram. Alternatively, Equation (10) may be used, redefining  $K$  as a function of the number of shear connectors between the point of zero moment and the section under consideration.



**Figure 8** Interaction between moment capacity and degree of shear connection in composite beams

### 5.4 Analysis of the composite section – negative (hogging) moment

In a composite section where the steel beam has equal flanges, the plastic moment resistance of the section under negative moment can be evaluated from the following formulae:

- Case 6:  $R_r < R_w$  (plastic neutral axis lies in web)

$$M_{nc} = M_s + R_r \left( \frac{D}{2} + D_r \right) - \frac{R_r^2 d}{R_v 4} \tag{11}$$

where  $R_r$  = the tensile resistance of the reinforcement within the effective breadth of the slab under negative moment

$D_r$  = the distance from the top of the steel beam to the centroid of the reinforcement

$R_v$  = the axial resistance of the web (over depth  $d$  between the flanges).

Alternatively, for slender or semi-compact webs,

where  $\frac{d}{t} > \frac{80\epsilon}{1 + \frac{R_r}{R_v}}$  and  $R_r < R_o$

$$M_{nc} = M_s + R_r \left( \frac{D}{2} + D_r \right) - \frac{R_r^2 + (R_v + R_r)(R_v + R_r - 2R_o) d}{R_v 4} \tag{12}$$

where  $R_o = 40t^2\epsilon p_y$   
 $\epsilon = (275/p_y)^{1/2}$

This second formula takes account of the neglected portion of the web in compression as in Figure 6(b).

- *Case 7:  $R_r > R_w$*  (plastic neutral axis lies in steel flange)

$$M_{nc} = R_s \frac{D}{2} + R_r D_r \quad \text{for } d \leq 40t\epsilon \quad (13)$$

$$\text{or } M_{nc} = (R_s - R_v + R_0) \frac{D}{2} + R_r D_r \quad \text{for } d \geq 40t\epsilon \quad (14)$$

Partial shear connection is not permitted in the negative moment region of the beam, and therefore selection of the practical amount of reinforcement that may be used in calculating  $R_r$  is dictated by the number of shear connectors placed in this region. Nominal slab reinforcement (i.e. welded mesh or bars of less than 10 mm diameter) should be neglected in calculating  $R_r$ . Hence, if no additional reinforcement is provided,  $M_{nc} = M_s$ .

## 5.5 Combined moment and shear

Continuous beams are subject to combined moment and shear at their supports. A design formula is given in *BS 5950:Part 3.1* for determining the reduced moment resistance of the composite section. In a haunched beam this would normally be critical in the beam section at the ends of the haunch.

The shear resistance of the web of a rolled section is  $P_v = 0.6p_y t D$ . If the applied shear force  $F_v$  exceeds  $0.5P_v$ , then a proportion of the shear area of the web is to be deducted in calculating the moment resistance. The reduced plastic moment resistance in the presence of shear is then given by:

$$M_{cv} = M_c - (M_c - M_f) \left( \frac{2F_v}{P_v} - 1 \right)^2 \quad (15)$$

where  $M_c$  = the plastic moment resistance of the composite section

$M_f$  = the moment resistance of the composite section having deducted the total shear area.

A similar expression may be derived for negative moment using  $M_c$  as calculated for  $M_{nc}$  in Section 5.4.

The above approach applies to all classes of sections provided  $P_v$  is defined as the lesser of the shear buckling or shear resistance of the section.

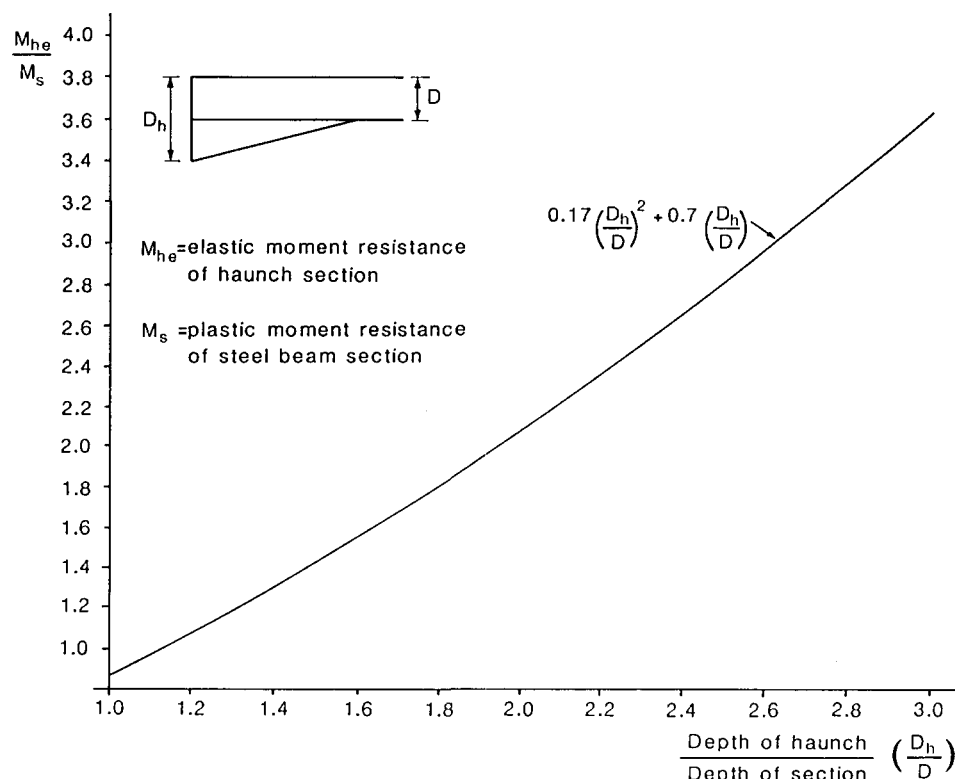
## 5.6 Transverse reinforcement

The longitudinal force transferred from the shear connectors is resisted by the concrete slab in shear and by the transverse reinforcement (orthogonal to the beam). The design for longitudinal shear is covered in *BS 5950:Part 3.1* and is not repeated here.

It is usually found that for secondary beams subject to uniform loading, standard mesh reinforcement provides sufficient transverse reinforcement provided the decking is properly anchored by shear connectors. For primary beams subject to point loads, concentration of shear connectors in the high shear zones is often such that additional transverse reinforcement is required. In extreme cases the shear resistance of the concrete may be exceeded leading to the use of deeper slabs.

## 6. ANALYSIS OF HAUNCHED SECTION

The bending resistance of the haunched section is evaluated elastically. The depth of the haunch is determined primarily to achieve an efficient moment connection to the column, and therefore a refined calculation of the capacity of the haunch is usually inappropriate. An approximate relationship between the elastic resistance of a haunched beam and the plastic resistance of the parent beam is shown in Figure 9. The haunch 'cutting' is made from the same beam section. If so, it is not necessary to check the adequacy of the haunch at intermediate sections provided the elastic resistance at its deepest section is adequate.



**Figure 9** Relationship between bending resistances of haunch and beam

It is apparent from Figure 9 that plastification may extend a short distance into the shallowest part of the haunch. This is acceptable because the tip of the haunch is to be laterally restrained where plastic hinges form, leading to greater rotational capacity of this part of the haunch. By ensuring that the deeper part of the haunch remains elastic, problems of instability can be treated by conventional theory (see Section 7.2).

The length of the haunch is selected to achieve the most efficient design of the beam and column. It is usually found that the haunch is 7% to 10% of the beam length in elastic global analysis and 5% to 7% in plastic hinge analysis (see Sections 4.1 and 4.2).

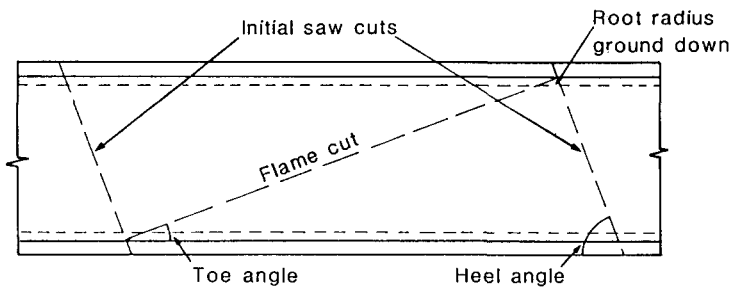
The moment is transferred to the column largely by the tensile and compressive forces in the outer flanges of the haunch. The force in the lower flange derives partly from the beam flange to 'cutting' flange weld at the tip of the haunch and partly from the beam to 'cutting' web weld along the haunch. It is a reasonable assumption that half of the compressive force is developed by each of these actions (see Figure 10). The force in the web-flange weld is assumed to act over a length not exceeding the beam depth. The flange to end plate welds should be designed to resist the forces transferred via the flanges.

A local vertical reaction is transferred to the beam web at the tip of the haunch and it is necessary to check the web strength in this region based on the force transferred through the flanges.



It is usually found that web stiffeners are required when the haunch taper exceeds approximately 15°. Web stiffeners are also required where the plastic moment resistance of the section is developed. This is also to ensure that transverse bending of the flanges does not occur at change of direction of the flange force. Full depth web stiffeners can also be used to provide lateral restraint to the bottom flange of the beam (see Section 7.3).

The haunch can also be designed compositely at internal columns. This is achieved by utilising the reinforcement (not including the mesh) in the slab. The effect of composite action is to reduce the haunch depth for the same moment. The design of the beam-column connection can potentially be simplified because the tensile forces in the bolts is reduced. It is suggested that, because of differences in the inherent deformation capacity between the connection and the high yield reinforcement in the slab, any reinforcement required to contribute to the flexural resistance of the composite section should be of a certain minimum amount to develop 'controlled cracking' in the concrete. This is typically equivalent to a minimum of 0.5% of the cross-sectional area of the slab.



FABRICATION CUTTING PLAN

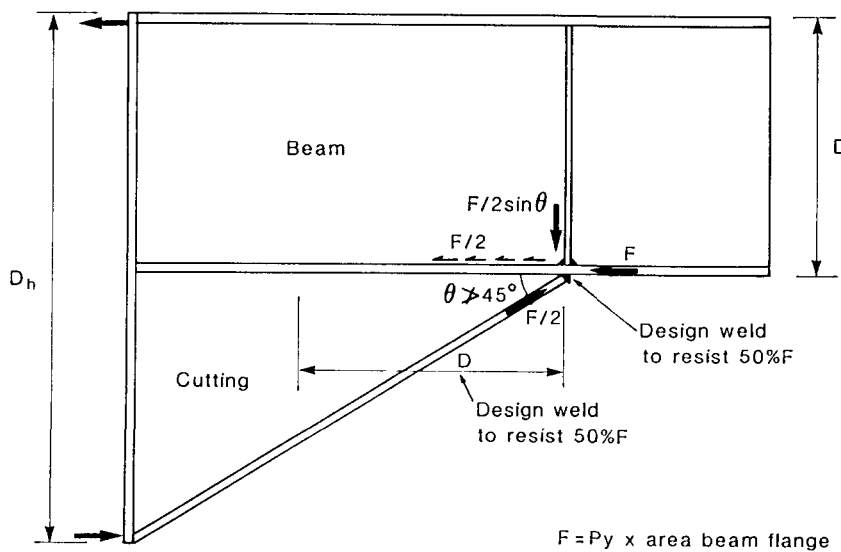


Figure 10 Detail of design of haunch



## 7. LATERAL STABILITY OF HAUNCHED COMPOSITE BEAMS

### 7.1 Lateral stability of non-composite beams

In the construction condition, loads are applied to the non-composite beams. These loads arise from the self-weight of the concrete and the structure and a uniform construction load of  $0.5 \text{ kN/m}^2$ .

The upper flanges of steel beams are assumed to be laterally restrained by profiled decking where the decking crosses the beams and is attached at regular intervals. In such cases the full plastic moment resistance of simply supported beams can be mobilized. Where the decking runs parallel to the beams little restraint is offered in the construction condition, and the buckling resistance of the beams is to be based on their slenderness between connections to secondary members.

In continuous beams, the situation is rather different. In the negative (hogging) moment region the compression flange is unrestrained, but the tension flange may be laterally restrained depending on the orientation of the decking. The effective slenderness of the beam in lateral torsional buckling is defined in *BS 5950:Part 1* as:

$$\lambda_{LT} = nuv_t \lambda \quad (16)$$

where  $u$  = the buckling parameter (typically 0.9 for universal beam sections)

$\lambda$  = the slenderness of the beam length between restraints

$n$  = the slenderness correction factor (for shape of bending moment diagram)

$v_t$  = the slenderness factor (including torsional stiffness and other effects).

For primary beams supporting secondary beams the stability of the primary beam is evaluated from its effective slenderness between the support and the first restraint (secondary beam), or between intermediate restraints in mid span. In either case, the lateral buckling moment determined from Clause 4.3.7 of *BS 5950:Part 1* can be compared to the applied moment.

For beams supporting decking, the tension flange is restrained when the beam is subject to negative moment. This reduces the effective slenderness by modifying the term  $v_t$ . A design formula for tension flange restraint is given in Appendix G3.3 of *BS 5950:Part 1*. Assuming the restraint acts at the top flange, then:

$$v_t = \frac{1}{\left[1 + \frac{1}{40} \left(\frac{\lambda}{x}\right)^2\right]^{1/2}} \quad (17)$$

where  $x$  = torsional index

In this case, the problem remains in defining the zone of the beam subject to lateral buckling. One approach is to define  $\lambda$  in terms of the beam length  $L$  and to select an appropriate value of  $n$  taking account of the shape of the bending moment diagram. An approximate formula is given in Clause G3.6 of *BS 5950:Part 1*. Where the zone subject to negative moment does not extend more than  $0.25L$  into the span, then  $n \approx (1/6)^{0.5} = 0.41$  for a beam of uniform section. This approach is very conservative and it is often found necessary to introduce additional lateral restraints.

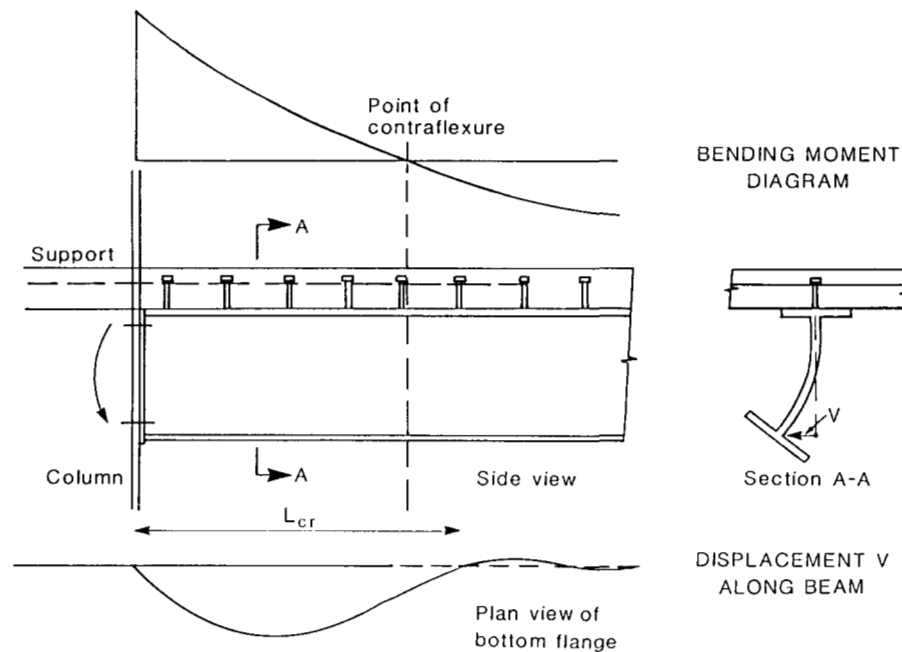
The treatment of haunched beams is also covered in Clause G3.6 of *BS 5950:Part 1*. There are two opposing effects. Firstly, the shape of the haunch closely follows the variation of bending moment and so flange stresses are likely to be constant in this region. Secondly, the 'third' flange provides additional resistance to lateral buckling. It is considered that the critical section for checking lateral buckling is the uniform section at the tip of the haunch. An effective value of  $n$  may be calculated using Clause G3.6. The factor  $c$  in Clause G3.2 is taken as unity for haunched beams with three flanges.

The effective slenderness of the beam is then used in Table 11 of *BS 5950:Part 1* to determine the buckling resistance of the section under negative moment. It should be noted that the torsional stiffness provided by the decking is not included in this analysis, but is clearly a beneficial factor.

The most onerous design condition, given the nature of the construction process, is when one span of a continuous beam is fully loaded and the adjacent span is unloaded. This means that more of the span is subject to negative moment. This can affect the design of the steel beam unless further lateral restraints to the lower compression flange are introduced such that the effective slenderness of the beam is reduced to the required value. Some examples of these restraints are given in the following Section.

## 7.2 Lateral stability of composite beams

In the composite condition the upper flanges of the steel beams are assumed to be laterally and torsionally restrained by the concrete or composite slab to which they are attached. In continuous beams, the lower compression flange is unrestrained except through the distortional stiffness of the cross-section. This is illustrated in Figure 11.



**Figure 11** Distortional buckling of composite beam in negative moment region

The effect of this distortional stiffness may be included by reducing the effective slenderness,  $\lambda_{LT}$ , of the beam (see Equation (16)). Despite the fact that local plastification may occur at the ends of the beam, this is not considered to affect the elastic mode of lateral instability of the beam in zones of rapidly reducing negative moment.

Using the approach of the preceding section the slenderness factor may be modified in the negative (hogging) moment region to include the lateral bending stiffness of the web, such that:

$$v_t = \frac{1}{\left[ 1 + \frac{1}{40} \left( \frac{\lambda}{x} \right)^2 + \frac{1}{16} \left( \frac{L_n}{D} \right)^3 \frac{I_w L_n}{I_y} \right]^{1/2}} \quad (18)$$

where  $L_n$  = the critical buckling length,  $L_{cr}$ , or the distance from the support to the first lateral restraint,  $L_1$  (whichever is the shorter)

$I_w$  = the second moment of area of the web per unit length =  $t^3/12$

$I_y$  = the second moment of area of the steel beam about its minor axis

$\lambda$  is based on  $L_n/r_y$ .

The formula may be derived by considering a beam subject to uniform negative moment and sinusoidal displacement of the lower flange of the beam between end supports. The upper flange is taken as being fixed in position and 'distortional buckling' occurs by out-of-plane bending of the web.

The critical buckling length corresponds to the half buckling wave length of a long span beam subject to distortional buckling of this form. This may be calculated by differentiation of Equation (18) to obtain the maximum value of  $\lambda_{LT}$ , such that:

$$L_{cr} = 3.74 I_y^{0.25} \left( \frac{D}{t} \right)^{0.75} \quad (19)$$

where  $D$  = the beam depth  
 $t$  = the web thickness.

In the absence of other lateral restraints, or when  $L_{cr} < L_1$ , insertion of Equation (19) reduces Equation (18) to:

$$v_t = \frac{1}{\left[ 2 + \frac{1}{40} \left( \frac{\lambda}{x} \right)^2 \right]^{1/2}} \quad (20)$$

The effective slenderness of the beam is obtained from Equation (16). Using the approximation that the area of each flange of UB sections is broadly equal to the web area, the term  $L_{cr}/I_y$  can be reduced to a function of the flange width  $B$  and thickness  $T$ .

The effective slenderness of the beam in the negative moment region (ignoring its torsional stiffness) now becomes:

$$\lambda_{LT} \approx 6.55 n u \left( \frac{T}{B} \right)^{0.25} \left( \frac{D}{t} \right)^{0.75} \quad (21)$$

$$\approx 3.0 n \left( \frac{D}{t} \right)^{0.75} \quad \text{for } \frac{B}{T} = 15 \quad (22)$$

This is very similar in form to the empirical formula presented by Johnson and Bradford<sup>(12)</sup>:

$$\lambda_{LT} = 3.4 \left( \frac{d}{t} \right)^{0.7} \quad (23)$$

where  $d$  = the web depth between flanges.

In many cases  $L_{cr}$  exceeds the length of the beam subject to negative moment. It is conservative in such cases to take  $n$  as 0.77 in Equation (22) (corresponding to a linear bending moment diagram). If  $L_{cr}$  (or  $L_1$  when using Equation (18)) is less than the length of the beam subject to negative moment then  $n$  is to be calculated from the moment ratio  $\beta$  at the location given by  $L_{cr}$  or  $L_1$ , as appropriate (see Table 16 of *BS 5950:Part 1*).

In haunched beams it would be normal practice to provide a lateral restraint at the tip of the haunch. This is obligatory where plastic hinges are developed. The stability of the haunch between the support and this lateral restraint is then checked using Clause 5.3.5 of *BS 5950:Part 1* (assuming that the lower flange is subject to uniform stress). This clause takes no account of the distortional restraint and is therefore very conservative.

The lateral stability of the beam in the zone from this restraint to mid span may be determined using Equations (16) and (18) or (22) depending on the magnitude of the critical buckling length  $L_{cr}$  and the slenderness correction factor  $n$ . In order to develop the plastic moment resistance of the section,  $\lambda_{LT}$  should be less than the limiting values in Tables 11 or 12 in *BS 5950:Part 1* (i.e. 34 for grade 43 steel and 30 for grade 50 steel).

In cases where elastic global analysis is used and the moment redistribution (based on gross section properties) is less than 10% it may be possible to avoid the use of lateral restraints at the tip of the haunch. Again, Equations (18) or (21) may be used.  $n$  should be calculated from Appendix G3.6 of *BS 5950:Part 1*, assuming uniform stress in the haunch zone. Conservatively,  $n$  may be taken as unity.

### 7.3 Restraint forces

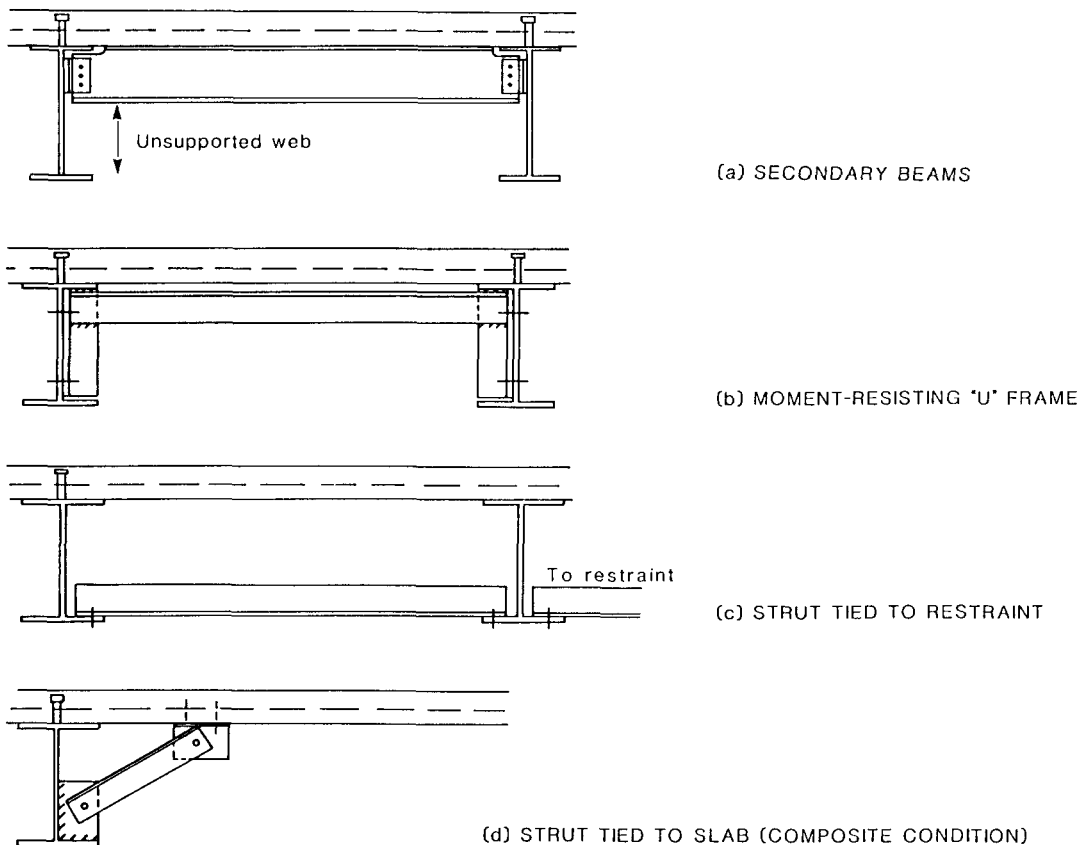
The above approach relates to the lateral stability of the member between restraints. It is also necessary to check that the lateral restraint is sufficiently strong. According to the amendment to *BS 5950:Part 1*, the total resistance offered to a compression flange by a continuous lateral restraint should not be less than 3% of the axial force in the flange. Hence, assuming that the restoring moment is 3% of the average applied moment distributed uniformly along the zone of the beam subject to lateral buckling, the lateral bending stress in the web can be easily calculated. This should be limited to  $p_y$ . This stress does not influence the calculation of the bending resistance of the section.

It is also necessary to check that the pull-out strength of the shear connectors is not exceeded. The lever arm is based on half the flange width. This is not usually critical to the design.

At a discrete restraint, such as at the tip of the haunch, the restraint force reduces to 2% of the flange force. However, this ignores the continuous restraint provided along the beam. Therefore, where both discrete and continuous restraints act together it is suggested that the discrete restraint is checked for 1% of the flange force at that location. The effective width for pull-out of the shear connectors may be conservatively taken as three times the slab depth (analogous to punching shear in concrete slabs).

Lateral restraints can be in the form of struts attached to the concrete slab or full depth web stiffeners. These only provide restraint in the composite condition. If the beam is also to be restrained in the construction stage, a 'goalpost' type support or positive restraint by secondary steelwork can be used. Various solutions are illustrated in Figure 12.

Where secondary beams frame into the webs of primary beams they offer lateral restraint to the unrestrained compression flange provided the distance from the point of attachment on the web to the compression flange is not excessive. As a rule of thumb, this distance, defining the unsupported length of web, should not exceed  $20t$ , where  $t$  is the web thickness.



**Figure 12** Different methods of providing the lateral restraint to bottom flange

## 8. SERVICEABILITY BEHAVIOUR OF COMPOSITE BEAMS

### 8.1 Elastic section properties

Elastic analysis is employed in establishing the serviceability performance of composite beams, or the strength of continuous beams subject to the effect of local instability. The important properties of the section are its section moduli and second moment of area. It is first necessary to determine the centroid (elastic neutral axis) of the transformed section by expressing the area of concrete in steel units. This is done by dividing the concrete area within the effective breadth of the slab  $B_e$  by an appropriate modular ratio  $\alpha_c$  (ratio of the elastic modulus of steel to concrete).

In unpropped construction, account should be taken of the stresses induced in the non-composite section as well as the stresses in the composite section. In elastic analysis, therefore, the sequence of construction is important. For elastic conditions to hold, extreme fibre stresses should be kept below their design values, and slip at the interface between the concrete and steel should be negligible.

The elastic section properties under positive moment can be evaluated from the transformed section. For buildings of normal usage,  $\alpha_c$  may be taken as 10 for normal weight concrete and 15 for lightweight concrete (density > 1750 kg/m<sup>3</sup>). The area of concrete within the profile depth is ignored (this is conservative where the decking troughs lie parallel to the beam). The concrete can usually be assumed to be uncracked under positive moment.

The elastic neutral axis depth  $y_e$  (below the upper surface of the slab) may be determined from the formula:

$$y_e = \frac{\frac{D_s - D_p}{2} + \alpha_c r \left( \frac{D}{2} + D_s \right)}{(1 + \alpha_c r)} \quad (24)$$

where  $r = \frac{A}{[(D_s - D_p) B_e]}$  and defines the relative proportion of the steel and concrete areas

$D_s$  = slab depth

$D_p$  = profile height

$A$  = the cross-section area of the beam of depth  $D$ .

The second moment of area of the uncracked composite section is:

$$I_{bc} = \frac{A(D + D_s + D_p)^2}{4(1 + \alpha_c r)} + \frac{B_e(D_s - D_p)^3}{12\alpha_c} + I \quad (25)$$

where  $I$  = the second moment of area of the steel section.

The section modulus for the steel in tension is:

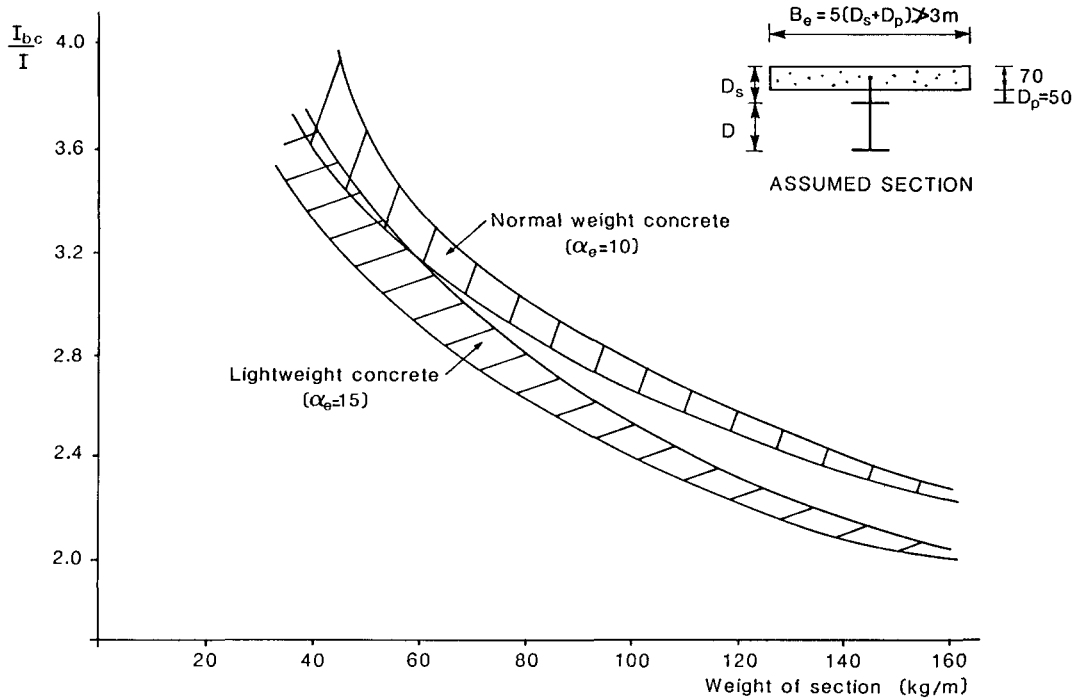
$$Z_t = \frac{I_{bc}}{D + D_s - y_e} \quad (26)$$

and for concrete in compression is:

$$Z_c = \frac{I_{bc} \alpha_c}{y_e} \quad (27)$$

The composite stiffness can be 2 to 3.5 times, and the elastic section modulus 1.3 to 1.7 times that of the I section alone for long span beams. The second moment of area of composite universal beam sections varies with section size as shown in Figure 13. This uses the same slab proportions as noted in Section 5.2. The difference between composite sections of light and normal weight concrete is shown.





**Figure 13** Ratio of second moment of area of composite section to that of steel section

## 8.2 Stresses in continuous and haunched composite beams

In design to *BS 5950:Part 3.1* it is necessary to check that the elastic stresses in the positive moment region do not exceed  $p_y$  in the bottom fibres of the steel section and  $0.5f_{cu}$  in the upper portion of the concrete slab. This is done to limit the effect of any local yielding on deflections. Section properties are determined as in Section 8.1. No account is taken of the effects of partial shear connection on the stresses in the beam.

The moments to be used in the calculation of stresses are the same as those used in the calculation of imposed load deflections. This is covered in Section 8.3. Stresses in the steel beam resulting from the self weight of the structure and the concrete slab should be added to those resulting from imposed load. It is not necessary to check the stresses in the negative moment region provided the approach in Section 8.3 is adopted.

## 8.3 Deflection of continuous and haunched composite beams

Elastic section properties, as described in Section 8.1, are used in establishing the deflection of composite beams. Uncracked section properties are considered to be appropriate for deflection calculations. The modular ratio depends on the duration of loading, but it is usually found that the section properties are relatively insensitive to the precise value of modular ratio. The effective breadth of the slab is the same as that used in evaluating the design strength of the beam (i.e. in mid span).

The deflection of a simple composite beam at service loads, where partial shear connection is used, can be calculated from formulae given in *BS 5950:Part 3.1*.

$$\delta'_c = \delta_c + 0.5(1 - K)(\delta_s - \delta_c) \quad \text{for propped beams} \quad (28)$$

$$\delta'_c = \delta_c + 0.3(1 - K)(\delta_s - \delta_c) \quad \text{for unpropped beams} \quad (29)$$

where  $\delta'_c$  = the beam deflection including the effects of slip  
 $\delta_c$  and  $\delta_s$  = the deflections of the composite and steel beam respectively at the appropriate serviceability load

$K$  = the degree of shear connection used in determining the plastic moment resistance of the beam in positive bending (Section 5.3).

The difference between the coefficients in these two formulae arises from the different shear connector forces and hence slip at serviceability loads in the two cases. These formulae are conservative with respect to other guidance<sup>(12)</sup> and are conservative when applied to continuous beams.

The effect of continuity in composite beams may be considered as follows. The imposed load deflection at mid span of a continuous beam under uniform load or symmetric point loads may be determined from the approximate formula:

$$\delta_{cc} = \delta'_c \left[ 1 - \frac{0.6(M_1 + M_2)}{M_0} \right] \quad (30)$$

where  $\delta'_c$  = the deflection of the simply supported composite beam for the same loading conditions, calculated from Equation (28) or (29)  
 $M_0$  = the maximum moment in a simply supported beam subject to the same loads  
 $M_1$  and  $M_2$  = the moments at the adjacent supports of the continuous or haunched beam span under consideration.

To determine appropriate values of  $M_1$  and  $M_2$ , an elastic analysis of a sub-frame may be carried out using the flexural stiffness of the uncracked section (ignoring the haunch).

In continuous beam design to *BS 5950:Part 3.1*, only one load case of imposed load applied equally to all spans is considered. The support moments are then redistributed by 30% for buildings of normal usage and 50% for buildings such as warehouses. This takes into account the effects of pattern loading and concrete cracking.

In haunched beams, there is a major difference in that framing into the columns reduces the influence of pattern loading. An equivalent approach to that in *BS 5950:Part 3.1* is to analyse the structure under the most probable loading at the serviceability limit state. For buildings of normal usage this may be taken as the full unfactored imposed loading on the span under consideration and one-third imposed load on the adjacent spans. For buildings or floors potentially subject to highly variable loads more extreme pattern loads should be considered. No further reduction in support moment is to be made when pattern loads of the above forms are taken into account.

In single-bay haunched beams, support moments based on elastic analysis of the frame using gross section properties should be reduced by 10% when using Equation (30). This takes into account the variation of stiffness in the negative moment region caused by concrete cracking (although partially offset by the haunch) and any possible over-estimate of column moment in the sub-frame approach.

For buildings subject to heavy or highly variable imposed loads (e.g. warehouses), there is a possibility of plastic rotation at the supports under repeated loading leading to greater imposed load deflections. This also affects the design of continuous beams designed by plastic hinge analysis, or where the effective redistribution of support moment in elastic analysis at the ultimate limit state exceeds 40% (using gross section properties in Table 1). In such cases a more detailed analysis should be carried out considering these effects as follows:

- Evaluate the support moments based on elastic analysis of the continuous beam under a first loading cycle of dead load and 80% imposed load (or 100% for semi-permanent load).
- Evaluate the excess moment where the above support moment exceeds the plastic moment resistance of the section under negative moment.
- The net support moments based on elastic analysis of the continuous beam under pattern load are to be reduced by the above excess moment.
- These support moments are input into Equation (30) to determine the imposed load deflection.
- The increased mid-span moment is to be used in checking stresses at serviceability.

## 8.4 Dynamic sensitivity

The use of longer span beams implies greater flexibility and although the in-service performance of composite beams and floors in existing buildings is good, the designer may be concerned about the susceptibility of the structure to vibration induced by activities performed within the building. The parameter commonly associated with this effect is the natural frequency of the floor or beams. The damping of vibration by a bare steel-composite structure is often low. However, when the building is occupied, damping is greatly increased.

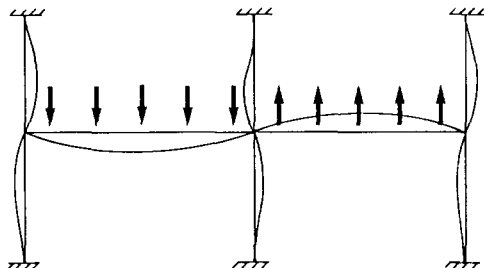
The natural frequency of a long span beam may be estimated from the simple formula:

$$f = \frac{18}{\sqrt{\delta_{sw}}} \text{ Hz} \quad (31)$$

where  $\delta_{sw}$  = the deflection of the composite beam assuming the self weight of the floor and other dead loads (including 10% imposed load) had been applied as a short duration load. This deflection may be reduced by 10% to take account of the increased dynamic stiffness of the beam above that used in imposed load calculations.

The natural frequency of a continuous beam is based on inertial forces (rather than static forces) which act in opposite senses on adjacent spans of a continuous beam (see Figure 14). In determining the deflection of the beam subject to the above loads, the gross uncracked section properties may be used.

The lower the natural frequency, the more the structure may respond dynamically to occupant-induced vibration. A limit of 4 Hz (cycles per second) is a commonly accepted lower bound to the natural frequency of the major elements of the structure<sup>(2)</sup>. Clearly, vibrating machinery or external vibration effects pose particular problems and in such cases it is often necessary to isolate the source of the vibration.



**Figure 14** Inertial forces used in natural frequency calculations

In practice, the mass of the structure is normally such that the exciting force is very small in comparison. This leads to the conclusion that long span structures may respond less in magnitude than light, short span structures. Guidance is given in the SCI publication<sup>(13)</sup>. If this guidance is followed then the natural frequency of the system (haunched beams, secondary beams and slab) could be reduced below 4 Hz. This depends on the 'response factor' which is itself a function of the area (and mass) of the floor participating in the response to an impulsive force.

Trial calculations suggest that for general office floors the effective mass of long span floor beams is sufficient to overcome any adverse effects of low natural frequency. A relaxation in the minimum natural frequency of haunched beams to 3.5 Hz is acceptable for beams longer than 15 m span (provided the natural frequency of the 'system' is not less than 3 Hz). This frequency limit of 3.5 Hz is often the controlling condition in the design of long span beams. However, it is recommended that the calculation method described in Reference (13) is followed for long span beams.



## 9. APPLICATION OF HAUNCHED BEAMS

The main principle in the design of a haunched composite beam is that framing into the columns reduces the design moment and deflection of the beams. It would not normally be practical to introduce large quantities of reinforcement into the slab as adequate moment transfer can usually be achieved through the haunch. Indeed, at edge columns it would be difficult to develop the required anchorage of the reinforcement, unless it is connected to the column (e.g. by anchorage into the concrete encasement).

The main use of haunched composite beams is therefore in two forms of structural grid:

### 1. *Closely spaced columns or mullions*

The haunched beams span between the columns and are directly loaded by the composite slab as in Figure 3(a). The spacing of the columns is therefore dictated by the span of the slab (3 to 3.6 m is typical for unpropped construction). The spacing of the shear connectors is influenced by spacing of the deck troughs.

### 2. *Wider spaced columns*

The haunched beams span between the columns, and in this case are loaded by secondary beams as in Figure 3(b). Consequently, the loading on the haunched beams is greater than in the previous case, leading to increased column moments. The depth of the secondary members should be less than the minimum depth of the haunched beam. Typical spans of the secondary beams are 5 to 8 m, depending on the spacing of the columns. The shear connectors are distributed to provide the required moment resistance along the haunched beam.

In multi-bay haunched frames it may be economic to reduce the moments applied to the perimeter columns by using a simple rather than haunched connection (see Figure 3(c)). This is advantageous for equal-bay frames not designed for sway resistance. In general, internal column moments tend to be relatively small because of the greater stiffness of the beams attached to the columns.

In situations where it is not possible for the haunched beam to frame directly into a column then, if necessary, moment continuity can be developed over a 'spine' beam using typical details as shown in Figure 3(d). These require more fabrication and also have the disadvantage of not developing the stiffening effect of the columns. Consequently, these beams will be heavier than those which connect directly to the columns.

An alternative to the traditional end plate connection is the welded haunch-to-column connection, which can be designed to mobilise the full moment resistance of the haunch. This form of connection is usually made in the factory by formation of 'column trees' comprising column lengths with welded haunch stubs. The main beams are then attached by bolting on site (see Figure 15). These bolted connections can be moment-resisting or simple shear-resisting connections, depending on the design of the beam.



**Figure 15** Use of welded column trees as haunched beams

Designs may be constrained further by the need to limit progressive collapse.

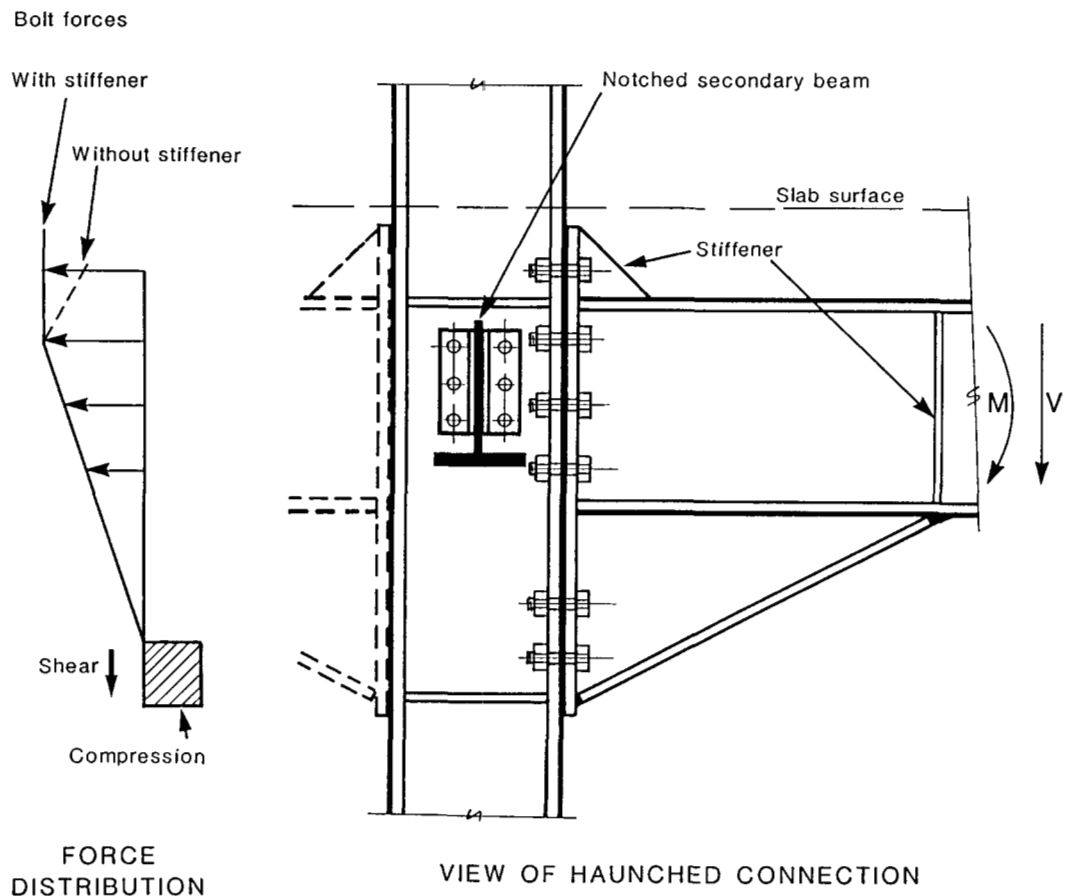
In Regulations terms, this is defined as 'robustness'. There are two approaches: design for the appropriate tying forces, or consider removal of individual beam or column members as 'key elements'. Beams supporting more than 70 m<sup>2</sup> of floor (usually primary beams in 2. above) are to be considered as key elements. In such cases it may be necessary to design the secondary beams to span between alternate primary beams (but with reduced load factors).

## 10. DESIGN OF HAUCHED CONNECTIONS

### 10.1 General principles

The depth of the haunch is primarily selected to minimise the number of bolts used and to avoid excessive local stiffening of the column. It is difficult to mobilise the full bending resistance of the haunch and typically the bending resistance of the connection would be 60 to 80% of the elastic resistance of the haunch.

It is normal practice to use a minimum of four bolts in the upper tensile zone of the connection and two in the compression zone. However, in many longer span beams with shallow haunches, it is often found necessary to use six or eight bolts in the tensile zone and four in the compression zone to transfer the required moment. The upper bolts are designed for tension, and the lower bolts for shear alone. Grade 8.8 M24 or larger bolts are required in long span beams.



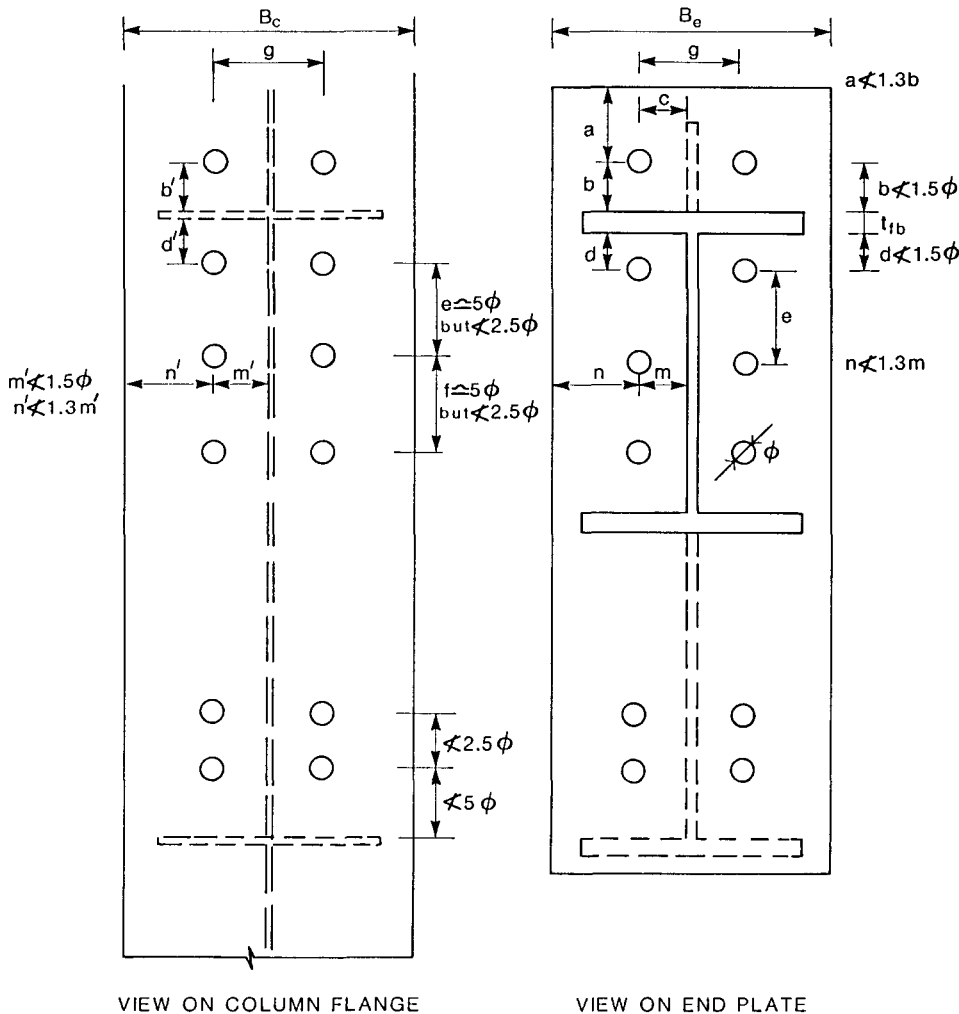
**Figure 16** *Distribution of bolt forces in haunched beam*

A typical haunched connection is shown in Figure 16. The use of extended end plates increases the bending resistance of the connection. The design of end plate connection is well established and economic design is based on 'yield-line' principles. This assumes that double curvature of the end plate is developed which results in additional prying forces in the bolts. Partial allowance for these additional forces is made in *BS 5950:Part 1*.

The designer normally has less control over the selection of the thickness of the column flange. To establish the maximum capacity of the column flange under the tensile effect of the bolts, it would be normal practice to carry out a further yield-line analysis of the flange. Design formulae have been developed for the standard cases of unstiffened and stiffened column flanges. (See Appendix A.)

## 10.2 Interim design procedure for end plate connections

1. Determine the design moment  $M$  and shear force  $V$  on the connection from Sections 4.1 and 4.2 making allowance for the increased moment as in Equation (3).
2. Select the connection type and the tentative bolt diameter  $\phi$  and layout based on practical spacings and edge distances. These are illustrated in Figure 17. The bolts would normally be located at  $1.5\phi$  to  $2\phi$  from their supporting elements and would be spaced at  $4\phi$  to  $5\phi$  apart down the web (see Point 10).



**Figure 17** Terminology used in yield-line analyses of end plate and column flanges

3. Determine the nominal force per bolt,  $F$ , based on elastic principles. This is done by first assuming a centre of rotation about the lower flange of the haunch and evaluating the second moment of area of the bolt group  $I_{bg}$  around this point. The force in each bolt is then  $M y_i / I_{bg}$  where  $y_i$  is the distance of each bolt from the centre of rotation. In the upper part of an extended plate the four upper bolts are assumed to resist equal force provided an additional stiffener is included.
4. Re-evaluate the centre of rotation assuming that there is a yielded zone of the beam in compression, such that the compressive resistance equals the total tensile force in the bolts. This requires trial and error. Re-assess the nominal bolt forces,  $F_i$ .
5. Determine the required number of bolts at the base of the haunch from the applied shear force divided by the number of bolts in this region.

6. Calculate the minimum thickness of the end plate based on yield-line analysis in terms of the plastic strength of the plate in double curvature. This depends on whether or not an additional stiffener is introduced at the top of the extended end plate.
7. Calculate the resistance of the column flange based on yield-line analysis. Reduce the plastic moment resistance of the flange taking account of axial stress  $\sigma$  in the flange according to the formula:

$$\left[ 1 - \left( \frac{\sigma}{p_y} \right)^2 \right]^{1/2} S p_y$$

where  $S$  = the flange plastic modulus ( $= T^2/4$ ).

This reduction applies only to the yield lines perpendicular to the axis of the column. Because of the magnitude of the forces applied to the column, it would be normal practice to introduce web stiffeners welded to the column flanges adjacent to the upper and lower flanges of the haunch.

8. If the column flange is unable to resist these forces, then the only practical solution is to increase the depth of the haunch (thereby reducing the bolt forces) or increase the column weight (and hence flange thickness).
9. Since double curvature is developed in the end plate and column flange, bolt forces are increased as a result of prying action. Provided the minimum edge distances are observed, prying forces may be typically 20% to 40% of the nominal bolt forces. In *BS 5950:Part 1*, the design capacities of the bolts include a 20% allowance for prying.

If the plastic resistance of the plates and flanges is fully utilized (i.e. using the plastic modulus  $S = T^2/4$ ) then it is suggested that the nominal bolt forces are increased by 20%. Alternatively, the bending resistance of the plates and flanges may be taken as  $S \leq 1.2Z$  (i.e.  $T^2/5$ ) when used with the design bolt strengths in *BS 5950:Part 1*.

These limitations improve the serviceability behaviour of the connection and avoid the risk of premature bolt failure.

The design of bolted connections of this type is covered by Horne and Morris<sup>(14)</sup> and Owens and Cheal<sup>(15)</sup>.

10. Check the strength of the welds between the end plate and the beam web and flange. This assumes a dispersion angle of 60° on either side of the bolt to the adjacent weld. The bolt spacing down the web is largely determined by the strength of the web and its welds.
11. Check the strength of the stiffeners attached to the column flange. Check the attachment of the secondary beams to the column web.



# 11. DESIGN OF COLUMNS

Columns are subject to axial forces and moment transferred from the haunched beams or as a result of sway action of the frame. External shear forces are generally small. For 'no-sway' frames, column moments may be calculated from elastic global analysis using the sub-frame as in Figure 4(b), or from plastic hinge analysis of the beams. In plastic hinge analysis or in elastic design where significant redistribution of moment occurs, column moments are increased as in Equation (3) of Section 4.2. The columns should be compact or plastic to *BS 5950:Part 1* to ensure that redistribution of moment arising from inaccuracies in the sub-frame approach does not lead to loss in strength of the columns.

The design of columns in no-sway frames may be treated as in Clause 4.8.3 of *BS 5950:Part 1*. When designing against buckling the influence of the shape of the bending moment diagram applied to the column may be included, taking account of the likelihood of differential loading on adjacent spans and floor levels.

As noted above, web stiffeners would normally be required to strengthen the column flange in the tensile zone and to prevent web buckling or yielding in the compressive zone adjacent to the haunched beams. The web panel should be checked for its shear strength between these stiffeners. The interaction between axial and shear stress should be considered. These stiffeners may affect the attachment of beams to the minor axis of the column.

It is normally found that edge columns, in particular, are considerably heavier than in simple construction. This is partly so that the thicknesses of the column flange is commensurate with the need to transfer a high moment and the thickness of the web to transfer a high local shear. Typically, this moment would be up to 70% of the bending resistance of the column, reducing to 50% in the lower storeys of a high rise building.

The design of columns in sway frames may be treated as in Clause 5.7.3 of *BS 5950:Part 1*. No redistribution of moments induced by lateral loads is permitted.

## 12. SCHEME DESIGN OF HAUNCHED COMPOSITE BEAMS

For Scheme Design, the following proportions of a haunched composite beam may be assumed:

- The ratio of overall length of the beam to overall depth (slab and beam but excluding haunch) should be in the range of 24 to 28 for most efficient design. Beams of these proportions when designed on 'strength' would usually satisfy 'serviceability'.
- For efficient design of haunched beams, the length of the end span should be between 75% and 115% of the length of the adjacent span.
- The length of the haunch is taken as 5% to 7% of the beam span if plastic hinge analysis is used or, typically, 7% to 10% if elastic global analysis is used.
- The total applied moment is calculated from the free span between the tips of the haunches. Assuming that the plastic moment resistance of the composite section is 1.5 times that of the steel section, the plastic moment resistance of the trial steel beam should not be less than 40% of the total applied moment (based on plastic hinge analysis).
- The column moment is calculated assuming a plastic hinge is present at the tip of the haunch. The column moment may be up to 70% of the bending resistance of the section in a building up to six storeys high.
- The column stiffness parameter  $I_c L / (I_{bc} h)$  should not be less than 0.3. (These terms are defined in Section 4.1).  $I_{bc}$  may be determined approximately from Figure 13, knowing the trial beam section.
- Grade 43 steel may be more economic than grade 50 in cases where the beam design is likely to be controlled by serviceability (i.e. for span:depth ratios greater than given above).
- The design of beams longer than 15 m may be controlled by the minimum natural frequency (3.5 Hz). This may necessitate use of lower span:depth ratios than those given above.
- The greatest depth of the haunch is taken as twice the depth of the steel section.
- The bending resistance of the bolted connection may be assumed to be 60% (for grade 50 steel) to 70% (for grade 43 steel) of the elastic moment resistance of the deepest haunch section.
- End plates are to be welded to the ends of the beam. These plates are approximately 20 to 30% thicker than the beam flange, and often similar to the bolt diameter.
- The bolt diameter is approximately equal to the end plate thickness. (M24 or M30 bolts are expected to be the preferred size.)
- For efficient design of the connection, the column flange thickness should not be less than 80% of the end plate thickness. This means that only the heavier rollings should be used for any serial size. The columns should also be compact or plastic<sup>(4)</sup>.

## 13. DESIGN PROCEDURES

1. Arrange the structural grid so that the haunched beams frame into the major axis of the columns. Avoid the use of adjacent beam spans of greatly differing lengths.
2. Carry out the Scheme Design as in Section 12. Select the trial beam and column sizes.
3. Determine the factored loads on the beams. Calculate the axial forces in the columns. Allow for the load reductions in *BS 6399*<sup>(16)</sup>, depending on loaded area (for beams) and number of storeys (for columns).
4. Evaluate the moment resistance of the composite sections in positive (sagging) bending as in Section 5.2. Initially, assume no additional reinforcement is placed in the slab, and hence the moment resistance in negative (hogging) bending is that of the trial section.
  - 5a. *Elastic global analysis (sway or no-sway frames)*
    - i. Calculate the composite stiffness of the trial beam section as in Section 8.1. Calculate the column stiffness.
    - ii. Establish the elastic moments in the frame assuming uniform uncracked stiffness of the composite beam. Use a sub-frame model as suggested in Section 4.1.
    - iii. Select the length of the haunch for efficient design of the beam.
    - iv. Check the adequacy of the beam by comparison with the moment resistance from 4 above. If required, redistribute the negative moment of the tip of the haunch in no sway frames up to the maximum percentages in Table 1, depending on the section classification. Maintain equilibrium by adding the redistributed moment to the positive moment.
    - v. Check the adequacy of the column section.
    - vi. Refine the design of the beams and columns, as necessary. Consider using additional reinforcement to enhance the negative moment resistance in multi-bay frames.
    - vii. Design the deepest part of the haunch and the beam-to-column connection for the elastic moment prior to redistribution. If plastic hinges are developed at the tip of the haunch, Equation (3) in Section 4.2 may be used.
  - 5b. *Plastic hinge analysis (no-sway frames)*
    - i. Select the haunch length. From the moment resistances in 4 above, determine the total load capacity of the beam by plastic hinge analysis as in Section 4.2. The steel section should be 'plastic' according to *BS 5950:Part 1*.
    - ii. Determine the moments transferred to the connection, haunch and column using Equation (3) in Section 4.2.
    - iii. Check the adequacy of these elements. Refine the design as necessary.
6. Check the design of the steel beam in the construction stage using the load factors in *BS 5950:Part 1*. This is mainly influenced by lateral buckling of the unrestrained flanges (see Section 7.1). Determine the stresses under self weight for use in serviceability calculations.
7. Select a practical distribution of shear connectors to satisfy the required degree of shear connection.
8. Determine the elastic section properties as in Section 8.1. Check the stresses under positive moment as follows:
  - i. Determine the support moments when the sub-frame is subject to unfactored imposed load on one span and one-third imposed load on adjacent spans (or zero imposed load in storage areas etc.).
  - ii. For single-bay frames, reduce the support moments by 10%. No reduction is required in multi-bay frames, provided the above load patterns are used.



- iii. Determine the net positive moment in the beam.
  - iv. Determine the stresses in the steel and the concrete, including the self-weight stresses from 6. Compare the total stresses with the limits in Section 8.2.
9. Determine the deflection of the beam under self weight and under imposed load. Use the moments calculated in 8. Where plastic hinge analysis is used or where the elastic redistribution of moment exceeds 30% at the ultimate limit state, it is necessary to calculate additional deflections as in Section 8.3.
  10. Check the natural frequency of the beam as in Section 8.4. For beams less than 15 m span limit the natural frequency to 4 Hz. (It may be possible to decrease this limit to 3.5 Hz for beams larger than 15 m span). Calculate the 'response factor', as in Reference (13).
  11. Check the lateral stability of the beam in the negative moment region as in Section 7.2. Introduce restraints to the tip of the haunch (obligatory in plastic hinge analysis).
  12. Check the conditions at the tip of the haunch. Introduce web stiffeners.
  13. Consider the 'robustness' requirements of *BS 5950:Part 1*. These include tying forces at connections or, if appropriate, design as key elements.
  14. Carry out final design of the connection as covered in Section 10.2. Introduce column stiffeners. Consider the connection of the secondary beams of the column webs.
  15. Check the slab for longitudinal shear.

## 14. PRACTICAL FEATURES AND DETAILS

Much of the design approach for haunched composite beams is based on existing portal frame technology<sup>(14)</sup>. It follows that the detailing of the steelwork largely reflects this method of construction. The critical elements are the beam-to-column connections, as they are required to transfer high moment and shear often in the presence of plastic hinges at the ends of the haunch.

The following points cover those that the designer should note, rather than detailed aspects of the fabrication. The aspects common to general composite construction are not covered.

### 14.1 Steel grade

The choice of steel grade is influenced by welding of the end plate and haunch and the thickness of the steel elements. Grade 43A or 50B are usually the preferred grades for the beams, using the design stresses in Table 3 of *BS 5950:Part 1*. Grade 50B is usually preferred for columns.

### 14.2 Welding

The thicknesses of the end plate and beam flange are such that these elements are likely to be welded using butt welds. The flange–flange weld at the end of the haunch may be butt or fillet welded. In thick joints it may be necessary to use low hydrogen welding rods or to use pre-heat to avoid hydrogen embrittlement. The end plate-to-web welds can be double-sided fillet welds.

Because of the lack of ductility of the welds it is vital that the load paths to the welds in the end plate connection are properly identified and the size of the welds checked accordingly.

### 14.3 Bolt spacing and grade

The spacing of the bolts is controlled by the requirements for bolt tightening, and to facilitate development of yield-line patterns in the column flange and end plate. The distance of the centre of the bolt to the supporting plate should not be less than  $1.5\phi$ , where  $\phi$  is the bolt diameter, nor 30 mm. The bolt spacing down the web would be typically  $4\phi$  to  $5\phi$ , as indicated in Figure 17.

The bolt grade should be grade 8.8, use of higher grades being inadvisable because of their low ductility. HSFG general grade bolts may be used. The bolt design may assume double curvature in the end plates and column flanges, subject to the limitations in Section 10.2.

### 14.4 Tolerances

The length of the haunched beams can be controlled accurately with modern fabrication methods. A tolerance of 0 to  $-2$  mm would normally be acceptable for end plate type connections. The use of shims is appropriate provided these are introduced to fill the gap between the column flange and the perimeter of the end plate. This is important because double curvature of these plate elements requires the development of points of contact at the edges of the plate. Normal bolt tolerances can be used.

## 14.5 Connection of secondary elements

Early consideration should be given to the practicalities of connection of the secondary beams to the primary beams and to the minor axis connection to the columns. This can influence the location of the column web stiffeners.

## 14.6 Precambering

In unpropped construction, deflections after concreting can be significant. If it is desired to finish the concrete surface 'level' the additional weight of concrete resulting from the deflection of the beam should be added or, alternatively, the beam precambered by an amount equal to the beam deflection under the self weight of the floor. Precambers of less than 25 mm are not usually practical.

## REFERENCES

1. LAWSON, R.M.  
Design of composite slabs and beams with steel decking  
Steel Construction Institute, 1989
2. LAWSON, R.M.  
Composite beams and slabs with profiled steel sheeting  
Construction Industry Research and Information Association, Report 99, 1983
3. JOHNSON, R.P.  
Composite structures of steel and concrete  
Granada, 1982
4. BRITISH STANDARDS INSTITUTION  
BS 5950:1985 Structural use of steelwork in building Part 1: Code of practice for design of simple and continuous construction : hot rolled sections  
BSI, 1985
5. BRITISH STANDARDS INSTITUTION  
BS 5950 Structural use of steelwork in building Part 3.1: Composite beams  
BSI, 1989 (draft for public comment)
6. LAWSON, R.M.  
Design for openings in the webs of composite beams  
Steel Construction Institute, 1987
7. KNOWLES, P.R.  
Design of castellated beams  
Steel Construction Institute, 1985
8. BRITISH STANDARDS INSTITUTION  
BS 5400:1979 Steel, concrete and composite bridges Part 5: Code of practice for the design of composite bridges  
BSI, 1979
9. BRETT, P.R., NETHERCOT, D.A. and OWENS, G.W.  
Continuous construction in steel for roofs and composite floors  
The Structural Engineer, Vol. 65A, No. 10, October 1987
10. COMMISSION OF THE EUROPEAN COMMUNITIES  
Eurocode 4: Common unified rules for composite steel and concrete structures  
Report EUR 9886 EN, 1985
11. AMERICAN INSTITUTE OF STEEL CONSTRUCTION  
Load resistance factor design manual of steel construction  
AISC, Chicago, 1986
12. BRADFORD, M.A. and JOHNSON, R.P.  
Inelastic buckling of composite bridge girders near supports  
Proc. Inst. Civil Engrs. Part 2, March 1987, **83**, pp143-159
13. WYATT, T.A.  
Design guide on the vibration of floors  
Steel Construction Institute, 1989
14. HORNE, M.R. and MORRIS, L.J.  
Plastic design of low rise frames  
Granada, 1982
15. OWENS, G.W. and CHEAL, B.D.  
Structural steelwork connections  
Butterworths, 1989
16. BRITISH STANDARDS INSTITUTION  
BS 6399:1984 Design loading for buildings Part 1: Code of practice for dead and imposed loads

## **Appendix A DESIGN EXAMPLE OF HAUNCHED COMPOSITE BEAM**

## *Commentary to calculation sheet*

### *Design of haunched composite beam*

*The design of a typical haunched composite beam and its supporting columns is considered. The building is 18m wide and this span is achieved by a single bay haunched beam. The building is taken as of 7 storeys height (for the purposes of designing the columns) and is braced against sway.*

*The beam and column arrangement is as shown opposite. The columns are spaced at 6m along the facade of the building and secondary composite beams spaced at 3m are supported by the primary haunched beams. The objective is to achieve the minimum structural depth subject to normal strength and serviceability criteria by transferring moments from the beams into the columns via the haunch and the beam to column connection.*

*In the Scheme Design, tentative beam and column sizes are established and these sizes are used in a full design check in the Final Design.*

*It is assumed that the "floor zone" is 1600mm deep and that 600mm depth below the beams can be used for servicing.*

*The floor slab is a composite slab which is designed to achieve 90 minutes fire resistance. From the Steel Construction Institute publication "Fire resistance of composite floors with steel decking" the minimum slab depth for lightweight concrete is 130mm when using a trapezoidal deck profile. Grade 30 concrete is used.*

*For purposes of this example, the frame to be analysed is that comprising the first floor haunched beam and adjacent columns. The column bases are conservatively assumed to be pinned. The same beam and column sizes may be used for the frames at other levels.*



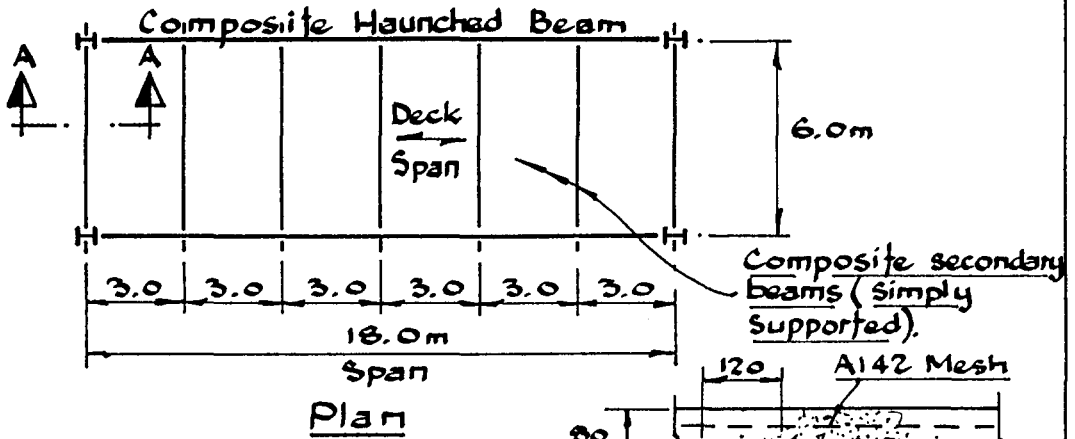
The Steel Construction Institute



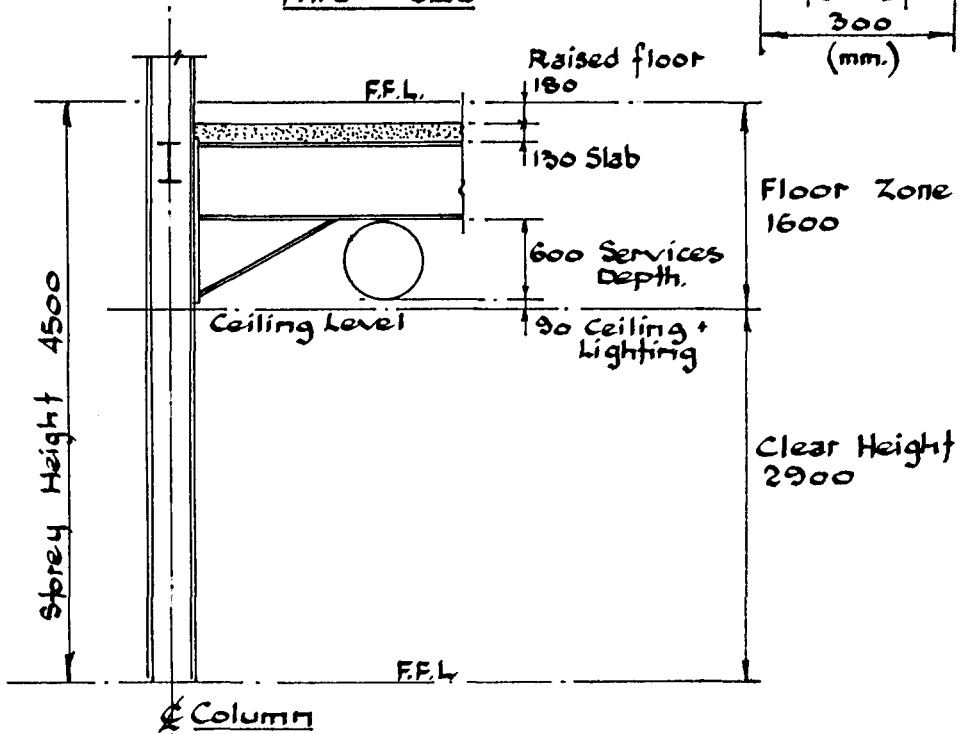
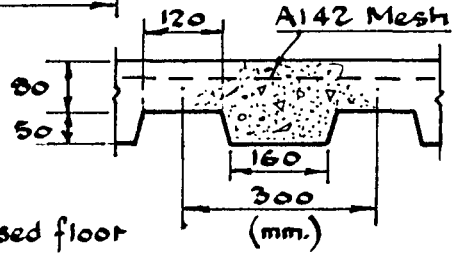
Silwood Park Ascot Berks SL5 7QN  
 Telephone: (0990) 23345  
 Fax: (0990) 22944 Telex: 846843

CALCULATION SHEET

Job No.	Sheet 1 of 31	Rev.
Job Title	APPENDIX A	
Subject	HAUNCHED BEAM DESIGN	
Client	Made by J.W.R.	Date April, 89
SCI	Checked by J.M.	Date April, 89



Typical Cross Section Thru ~ Slab



Enlarged Section A~A

<p><b>The Steel Construction Institute</b></p> <p>Silwood Park Ascot Berks SL5 7QN          Telephone: (0990) 23345          Fax: (0990) 22944 Telex: 846843</p> <p><b>CALCULATION SHEET</b></p>	Job No.	Sheet <b>2</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.V.R</i>	Date <i>April, 89</i>
	Checked by <i>B.M</i>	Date <i>April, 89</i>	

Loading Data

	<b>kN/m<sup>2</sup></b>
Slab weight	2.1
Construction Load	0.5
Main beams self wt.	1.5 kN/m
Secondary beams self wt.	0.25 kN/m
Raised Floor	0.2
Ceiling and Services	0.5
Imposed	
Occupancy	4.0
Partitions	<u>1.0</u>
	5.0
Roof	1.5

Scheme Design

Secondary beams ~ simply supported;  
 from a complete design check

Use ~ 254x102x25 UB. Grade 50 ~

Main Rigid Frames.  
 Consider Ultimate in ~ service loading  
 Main beam loading ~  
 Loaded area per beam =  $(18 - 2 \times 3/2) \times 6 = 90 \text{ m}^2$   
 $\therefore$  reduce imposed load by 9%

Imposed = 5.0 kN/m<sup>2</sup>  
 reduce by 9%       $\frac{-0.45}{4.55}$       Say 4.5 kN/m<sup>2</sup>

Dead =  $(2.1 + 0.5 + 0.2) = 2.8 \text{ kN/m}^2$   
 Secondary beam      =  $\frac{0.08}{2.88}$       Say 2.9 kN/m<sup>2</sup>

Factored Design Load per secondary beam

Imposed =  $1.6 \times 4.5 \times 3.0 \times 6.0 = 130 \text{ kN}$   
 Dead =  $1.4 \times 2.9 \times 3.0 \times 6.0 = 73 \text{ kN}$   
203 kN

Factored self wt. of main beam.

=  $1.4 \times 1.5 \text{ kN/m}$       = 2.1 kN/m

<p><b>The Steel Construction Institute</b></p> <p>Silwood Park Ascot Berks SL5 7QN          Telephone: (0990) 23345          Fax: (0990) 22944 Telex: 846843</p> <p><b>CALCULATION SHEET</b></p>	Job No.	Sheet <b>3</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>ZM</i>	Date <i>Apr 7, 89</i>	

Consider a substitute frame as shown ~

Beam size

Guess beam size based on a span/depth ratio of 26

$$\therefore 26 = \frac{18000}{(D+130)} \quad \therefore D = 562 \text{ mm.}$$

Try section 610 x 229 x 125 UB Grade 43  
 section classification ~ plastic  
 Plastic moment of resistance (non-composite) = 975 kN.m.

Column size

Guess column size by assuming a subframe as follows: (assume haunch length = 0.05L = 0.9 m.)

$$M_{col} = \frac{0.05L \cdot V + M_s}{2}$$

$$= \frac{0.05 \times 18 (5 \times 203 / 2) + 975}{2}$$

$$= 716 \text{ kN.m}$$

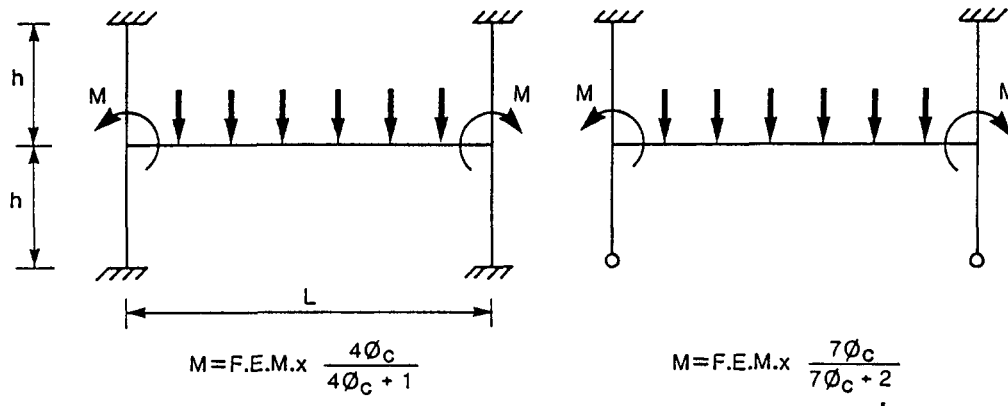
Assume  $M_{col}$  is 70% of column moment resistance ~

$$\therefore \text{Required moment of resistance} = \frac{716}{0.7} = 1023 \text{ kN.m.}$$

## Commentary to calculation sheet

The Final Design of the composite beams is carried out on the basis of elastic global moments. In the construction stage or, alternatively, in-service stage, moments can be evaluated from moment distribution throughout a sub-frame. The sub-frame is as suggested in BS5950:Part 1, (Figure 11(a)). Different cases of moments in single bay frames are considered below. F.E.M. is the fixed-end moment when the beam is subject to its applied load, assuming no rotation of the column.  $M$  is the support (negative) moment in the beam. This is distributed to the columns in accordance with the stiffness of the members above or below the floor.

For this example, the case considered is that of a single bay frame with pinned bases. This represents the first floor frame. When analysing the sub-frame the ends of the columns remote from the frame under consideration are taken as fixed (except at the foundation level).



<p><b>The Steel Construction Institute</b></p> <p>Silwood Park Ascot Berks SL5 7QN          Telephone: (0990) 23345          Fax: (0990) 22944 Telex: 846843</p> <p><b>CALCULATION SHEET</b></p>	Job No.	Sheet <b>4</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI.</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>Q.M.</i>	Date <i>April, 89</i>	

Try **305x305x198UC Grade 50**  
 section classification ~ Plastic  
 Plastic moment of resistance = 1170 kN.m.

**FINAL DESIGN**


Construction stage (non composite).  
 Loading ~

- Dead ~ slab (see sheet 2) = 2.1 kN/m<sup>2</sup>
- Dead load from secondary beam = (2.1 x 3.0 + 0.25) 6.0 = 39.0 kN.
- Construction Load = 0.5 kN/m<sup>2</sup>
- Construction Load from secondary beam = 0.5 x 3.0 x 6.0 = 9.0 kN.

Check Ultimate Limit state  
 Ultimate design load from secondary beam = 1.4 x 39 + 1.6 x 9.0 = 69.0 kN.

Using the substitute frame ~

**Elevation**

<b>The Steel Construction Institute</b>  Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843 <b>CALCULATION SHEET</b>	Job No.	Sheet <b>5</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>J.M.</i>	Date <i>April, 89</i>	

$$F.E.M. = \frac{69 \times 18 (36-1)}{12 \times 6} + \frac{1.4 \times 1.5 \times 18^2}{12} = 660 \text{ kN.m.}$$

Column stiffness parameter  $\phi_c = \frac{I_{COL} \cdot L}{I_{BEAM} \cdot h}$

$\therefore$  at the construction stage  $\phi_c = \frac{50800 \times 18}{98600 \times 4.5} = 2.06$

Hence from figure on Commentary page

$$M_{support} = \frac{7\phi_c}{(7\phi_c + 2)} \cdot F.E.M. = \frac{7 \times 2.06}{(7 \times 2.06 + 2)} \times 660$$

$$= \underline{580 \text{ kN.m.}}$$

The maximum free bending moment

$$= \frac{69 \times 6 \times 18}{8} + \frac{1.4 \times 1.5 \times 18^2}{8} = \underline{1017 \text{ kN.m.}}$$

The maximum positive (sagging) bending moment

$$= 1017 - 580 = \underline{437 \text{ kN.m.}}$$

The maximum negative (hogging) bending moment at the haunch tip: ~

haunch length = 900 mm.

Support reaction =  $(69 \times 5 + 1.4 \times 1.5 \times 18) \times 0.5 = 191 \text{ kN.}$

Haunch tip moment =  $580 + \frac{1.4 \times 1.5 \times 0.9^2}{2} - 191 \times 0.9$


$$= \underline{410 \text{ kN.m.}}$$

Since section  $M_s = 975 \text{ kN.m} > 437$  and  $410 \text{ kN.m.}$

Section is Adequate

For stability check, see later (sheet 21).



<p><b>The Steel Construction Institute</b></p>  <p>Silwood Park Ascot Berks SL5 7QN          Telephone: (0990) 23345          Fax: (0990) 22944 Telex: 846843</p> <p><b>CALCULATION SHEET</b></p>	Job No.	Sheet <b>6</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April 89</i>
	Checked by <i>LM</i>	Date <i>April, 89</i>	

Main Beam ~ Ultimate in ~ Service condition (Composite).  
Section 610 × 229 × 125 UB. Grade 43

Consider Elastic method of design for global moments

Elastic Composite Properties  
 depth from top of slab to neutral axis,  $y_e$

$$y_e = \frac{(D_s - D_p) 0.5 + \kappa_e \cdot r \left( \frac{D}{2} + D_s \right)}{(1 + \kappa_e \cdot r)}$$

where  $r = A / [(D_s - D_p) B_e]$

Effective breadth,  $B_e = 0.7 \times \frac{18000}{4} = 3150 \text{ mm.}$

modular ratio = 15 for Lightweight concrete.

$$r = \frac{160 \times 10^3}{[(130 - 50) 3150]} = 0.0635$$

$$y_e = \frac{(130 - 50) 0.5 + 15 \times 0.0635 \left( \frac{612}{2} + 130 \right)}{(1 + 15 \times 0.0635)} = 233 \text{ mm.}$$

the second moment of area  $I_{bc}$  is found from

$$I_{bc} = \frac{A (D + D_s + D_p)^2}{4 (1 + \kappa_e \cdot r)} + \frac{B_e (D_s - D_p)^3}{12 \kappa_e} + I$$

$$= \frac{160 \times 10^3 (612 + 130 + 50)^2}{4 (1 + 15 \times 0.0635) 10^4} + \frac{3150 (130 - 50)^3}{12 \times 15 \times 10^4} + 98600$$

$$= \underline{228000 \text{ cm}^4}$$

The Steel Construction Institute



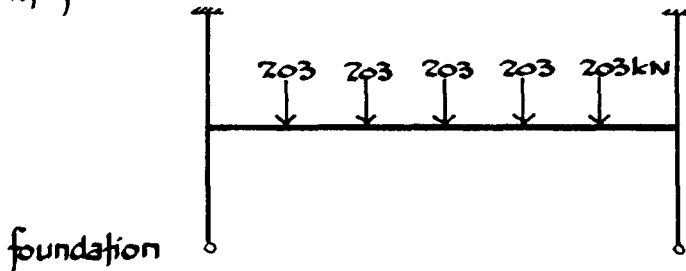
Silwood Park Ascot Berks SL5 7QN  
Telephone: (0990) 23345  
Fax: (0990) 22944 Telex: 846843

CALCULATION SHEET

Job No.	Sheet 7 of 31	Rev.
Job Title	APPENDIX A	
Subject	HAUNCHED BEAM DESIGN	
Client	Made by J.W.R.	Date April, 89
SCI	Checked by KM	Date April, 89

Global Moments  
Using substitute frame ~

1st. floor



203 kN = factored design load for each secondary beam.  
(see sheet 2).

Beam ~ 610 x 229 x 125 UB. Grade 43  
Column ~ 305 x 305 x 198 UC. Grade 50

$$\phi_c = \frac{I_{col} \cdot L}{I_{bc} \cdot h} = \frac{50800 \times 18}{228000 \times 4.5} = 0.89$$

$$FEM. = \frac{203 \times 18 (36-1)}{12 \times 6} + \frac{2.1 \times 18^2}{12} = 1833 \text{ kN.m.}$$

$$M_{support} = \frac{7\phi_c}{(7\phi_c + 2)} \cdot FEM. = \frac{7 \times 0.89}{7 \times 0.89 + 2} \times 1833 = 1388 \text{ kN.m.}$$

Maximum 'free' bending moment,  $M_0$


$$= \frac{6 \times 203 \times 18}{8} + \frac{2.1 \times 18^2}{8} = 2826 \text{ kN.m.}$$

Maximum positive (sagging) moment

$$= 2826 - 1388 = 1438 \text{ kN.m.}$$

End reaction

$$= \frac{(5 \times 203 + 2.1 \times 18)}{2} = 526 \text{ kN.}$$

<b>The Steel Construction Institute</b>  Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843 <b>CALCULATION SHEET</b>	Job No.	Sheet <b>8</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>J.M.</i>	Date <i>April, 89</i>	

Composite Section  
Resistance moment for full interaction

Compression Resistance of slab,  $R_c$

$$= 0.45 f_{cu} (D_s - D_p) B_e \quad \dots (4)$$

$$= 0.45 \times 30 \frac{(130 - 50) 3150}{10^3} = 3402 \text{ kN.}$$

Tensile Resistance of steel section,  $R_s$

$$= A \cdot p_y = \frac{160 \times 10^2 \times 265}{10^3} = 4240 \text{ kN.}$$

As  $R_s > R_c$  plastic neutral axis lies in steel flange

Tensile Resistance of flange,  $R_f$

$$= B \cdot T \cdot p_y = \frac{229 \times 19.6 \times 265}{10^3} = 1189 \text{ kN.}$$

Tensile Resistance of web,  $R_w$ .

$$= R_s - 2R_f = 4240 - 2 \times 1189 = 1862 \text{ kN.}$$

since  $R_s > R_c > R_w$ , plastic neutral axis lies in steel flange, then the moment of resistance for the composite section is given by,  $M_{pc}$


$$= \frac{R_s \cdot D}{2} + R_c \left( \frac{D_s + D_p}{2} \right) - \frac{(R_s - R_c)^2}{R_f} \cdot \frac{T}{4} \quad \dots (6)$$

$$= \frac{4240 \times 612}{2 \times 10^3} + \frac{3402 (130 + 50)}{10^3 \times 2} - \frac{(4240 - 3402)^2}{1189} \cdot \frac{19.6}{10^3 \times 4}$$

$$= 1601 \text{ kN.m.}$$

But since 1601 > 1438 kN.m

$\therefore$  Satisfactory.

<b>The Steel Construction Institute</b>  Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843 <b>CALCULATION SHEET</b>	Job No.	Sheet <b>9</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>J.M.</i>	Date <i>April, 89</i>	

Negative (hogging) moment of resistance  
 ~ No reinforcement is to be provided

Moment of Resistance = that of steel section alone  
 check interaction with shear - BS.5950: Part 1  
 most onerous section occurs at haunch tip  
 assume shear force  $\approx$  support reaction

$\therefore F_v = 526 \text{ kN.}$

$R_v = 0.6 p_y. A_v = \frac{0.6 \times 265 \times 11.9 \times 612}{10^3} = 1158 \text{ kN.}$

$0.6 R_v = 0.6 \times 1158 = 695 \text{ kN} > 526 \text{ kN.}$

since  $F_v < 0.6 R_v$  no reduction in moment resistance

$\therefore M_{pc} = M_s = \underline{975 \text{ kN.m.}}$

A haunch length of 1200 mm. is to be used  
 ~ refer to BM. diagram.

Redistribution of Moments

From sheet 5, since the haunch tip moment  $< M_s$   
 no redistribution can be considered

$\therefore$  Global Moments remain unchanged.

Degree of Shear Connection

Connector strength for positive moments  
 $Q_p = 0.8 Q_k. r_p$

$Q_k$  for 19mm. dia. stud, 95mm (LAW) high and  $f_{cu} = 30 \text{ N/mm}^2$   
 $= 100 \text{ kN} \times 90\%$  (for lightweight concrete)  $= 90 \text{ kN}$

for ribs parallel to the beam,

$b_r/D_p = \frac{\text{average trough width}}{\text{profile depth}} = \frac{170}{50} = 3.4 > 1.5$

$\therefore$  Take  $r_p = 1$  and  $Q_p = 0.8 \times 90 \times 1 = 72 \text{ kN}$

Extent of positive moment region = Span -  $2x$

<p><b>The Steel Construction Institute</b></p> <p>Silwood Park Ascot Berks SL5 7QN          Telephone: (0990) 23345          Fax: (0990) 22944 Telex: 846843</p> <p><b>CALCULATION SHEET</b></p>	Job No.	Sheet <b>10</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>J.M.</i>	Date <i>April, 89</i>	

where  $x$  = distance to point of contraflexure from support  
 $x$  is given by:-

$$1388 - 526x = 0 \quad \therefore x = 2.639 \text{ m.}$$

length of half positive moment region  
 $= \frac{18 - 2 \times 2.639}{2} = 6.361 \text{ m.}$

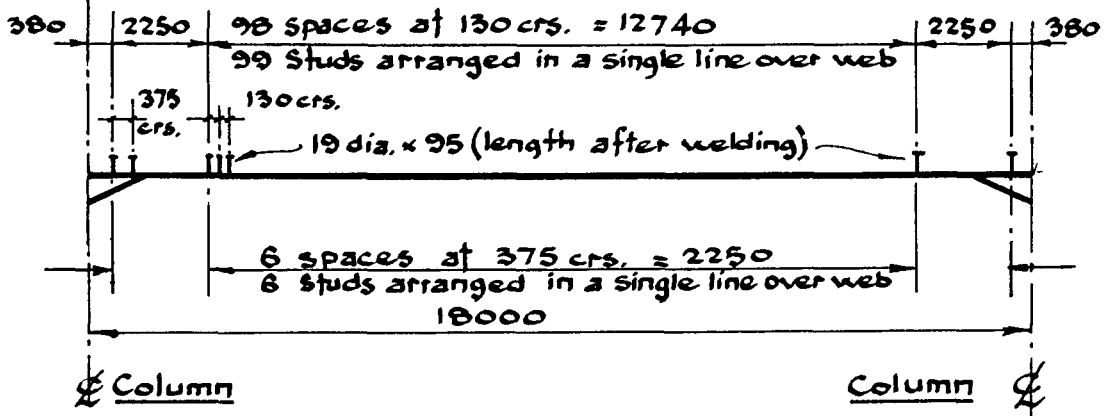
Number of connectors required for full interaction  
 $= \frac{R_c}{72} = \frac{3402}{72} = 47.3$ , say 48 Try equal spacing =  $\frac{6361}{48} = 133 \text{ mm.}$

Provide connectors at 130mm. spacing in positive moment region.

Check minimum spacing =  $6d = 6 \times 19 = 114 \text{ mm} < 130 \text{ mm. OK}$

Maximum spacing in negative moment region  
 $= 600 \text{ mm. or } 4D_s = 4 \times 130 = 520 \text{ mm.}$

by inspection, provide connectors at 375mm. in negative moment region



**Stud Connector Layout**

## ***Commentary to calculation sheet***

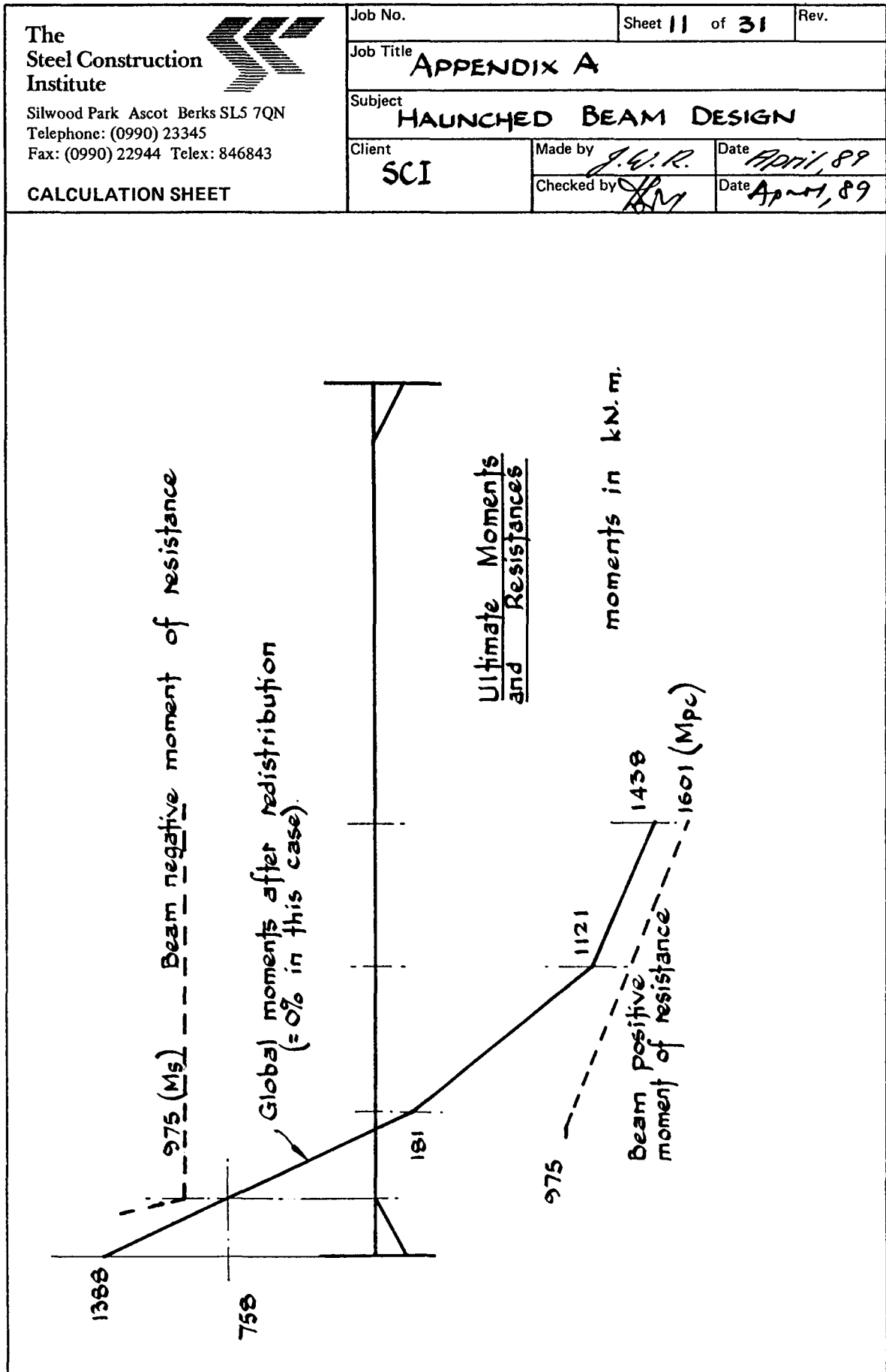
*The elastic bending moment diagram for the haunched beam subject to design (factored) loading is as shown opposite.*

*Also indicated are the moment resistances of the composite section in positive (sagging) bending and of the non-composite section in negative (hogging) bending. The haunch capacity is obtained from Figure 9. However, the limiting condition for the design of the haunch is normally the ability of the connection to transfer the required moment.*

*The length of the haunch is selected so that the moment resistance of the steel section exceeds the applied moment. Therefore, no redistribution of moment is required. The haunch length (defined to the centre-line of the column) is taken as 1200mm. Hence, the actual haunch "cutting" length is 1000mm (allowing for half the column width and end plate thickness).*



[Discuss me ...](#)



## *Commentary to calculation sheet*

### *Column Design*

*The calculation of the moments and forces in the columns has been simplified for the purposes of this example. In principle, checks are to be made on the local capacity and on the overall buckling capacity of the columns. Appropriate load combinations are to be considered to have the worst effect on a given column section.*

*The overall buckling check is influenced by the shape of the bending moment along the column, whereas the local capacity check is dependent only on the magnitude of the moment. Both are considered in combination with axial load.*

*To calculate the axial load, the reduction in imposed load above the floor under consideration is included. According to BS6399:Part 1 Table 2, this reduction is 40% for the ground or first floor column of a 7 storey building. A cladding load of 10 kN/m is also included when calculating the axial forces in these columns which are at the perimeter of the building.*

*The sub-frame considered is as recommended in BS5950:Part 1 (Figure 11(b)). The worst load combination for the ground-first floor column was found to be imposed load on all floors except the second floor. The fixed end moments (FEM) in all the beams have been calculated considering the loads applied to the non-composite and composite section respectively.*

<p><b>The Steel Construction Institute</b></p> <p>Silwood Park Ascot Berks SL5 7QN          Telephone: (0990) 23345          Fax: (0990) 22944 Telex: 846843</p> <p><b>CALCULATION SHEET</b></p>	Job No.	Sheet <b>12</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>J.M.</i>	Date <i>April 89</i>	

**Column Design**  
 Consider ground floor/1st. floor subframe  
 Design Case 1 ~ largest column moment + associated axial load  
 Loading ~ 2nd. floor dead only other floors fully loaded  
 Unfactored axial load in column just above first floor  
 Dead = 1395 kN.  
 Imposed = 729 kN.  
 Design axial load =  $1.4 \times 1395 + 1.6 \times 729 = 3119$  kN.

The loading must now be separated into that on the non-composite frame and that on the composite frame as follows ~

**Non-Composite Frame**

**Composite Frame**

The Design F.E.M. at 1st. and 2nd Floors due to the slab weight + beam self weight was found to be 458 kN.m.

The Design F.E.M. at second floor due to superdead only was found to be 154 kN.m.

The Design F.E.M. at 1st. floor due to full dead + imposed = 1833 kN.m.

∴ Design F.E.M. due to Superdead + Imposed  
 =  $1833 - 458 = 1375$  kN.m.

## Commentary to calculation sheet

### Column Design - continued

*The moment distribution in the sub-frames is carried out in terms of the relative stiffness of the columns and beams in the non-composite and composite states. This has not been included here. The moments for the two cases are added and the final distribution is shown opposite.*

*The greatest moment is in the column above the first floor and is 714 kNm. The axial load in this column is as calculated previously and is 3119 kN.*

*From the Steel Construction Institute publication 'Guide to BS5950:Part 1:1985 Volume 1':page 302, the reduced moment capacity of the column is 875 kNm when subject to the above axial load.*

*When considering overall buckling, the worst case for design is the column below the first floor. The load case is the same as that considered for the local capacity check. The bending moment in the column decreases to zero at the pinned base, and the effect of this is taken into account by an equivalent moment factor  $m$ . In this case  $m$  is obtained from Table 18 of BS5950:Part 1 and is taken as 0.57.*

*The permissible bending and axial stress are obtained from the slenderness of the column. The effective length of the column is conservatively taken as the actual column length. Again the buckling moment is obtained from page 302 of the above SCI publication. The axial capacity  $A_g p_y$  is also obtained from page 214 of this publication.*

*The further load case of full loading on all spans would slightly increase the axial load on the column, but would significantly reduce the applied moment and would cause a reduction in the moment factor  $m$ . Therefore, it is usually a less onerous design case for both the local capacity and overall buckling checks.*

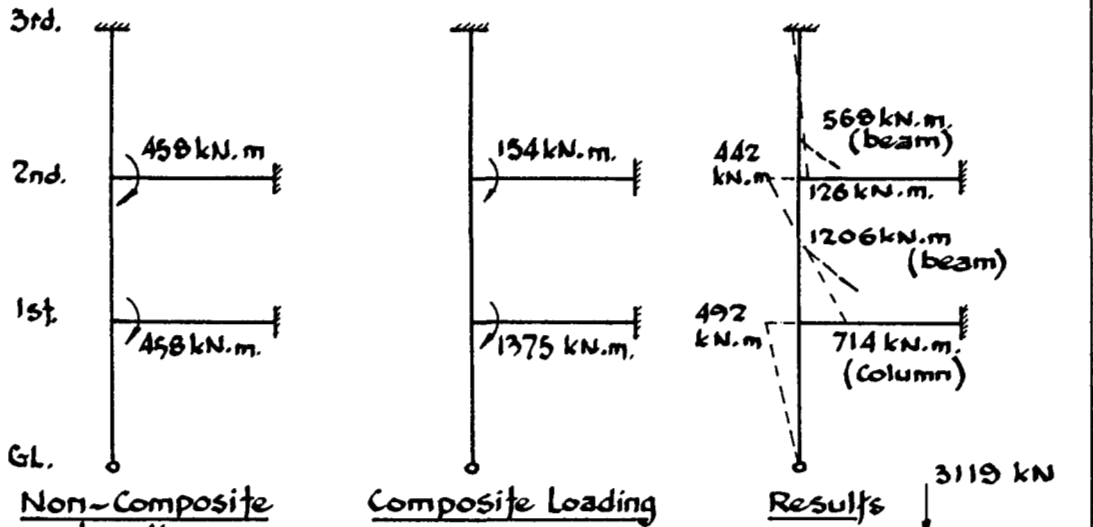
The Steel Construction Institute



Silwood Park Ascot Berks SL5 7QN  
 Telephone: (0990) 23345  
 Fax: (0990) 22944 Telex: 846843

CALCULATION SHEET

Job No.	Sheet 13 of 31	Rev.
Job Title	APPENDIX A	
Subject	HAUNCHED BEAM DESIGN	
Client	Made by J.W.R.	Date April, 89
SCI	Checked by [Signature]	Date April, 89



Section check ~ 1st. floor column

Try 305 x 305 x 198 UC Grade 50

Local capacity check ~

$$\frac{F}{P_z} = \frac{3119}{8570} = 0.36, \text{ from published tables}$$

$$M_{rx} \approx 875 > 714 \text{ kN.m} \therefore \text{Section Satisfactory}$$

Design Case 2 ~ most onerous moment shape + associated axial load.

Consider the ground floor column

Loading ~ as before, 2nd. floor dead load only  
 other floors fully loaded

Unfactored axial load in ground floor column

$$\begin{aligned} \text{Dead} &= 1395 + 244 = 1639 \text{ kN.} \\ \text{Imposed} &= (81 + 5 \times 270 \times 60\%) = 891 \text{ kN.} \end{aligned}$$

$$\text{Design axial load} = 1.4 \times 1639 + 1.6 \times 891 = 3720 \text{ kN.}$$

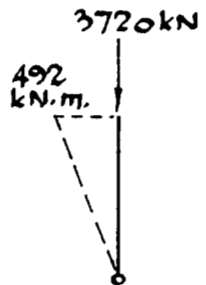
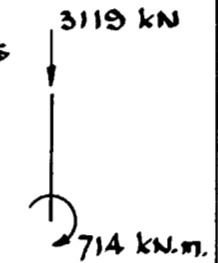
Use the same substitute frames for simplicity


Section check ~ Ground floor column

Overall Buckling check

$$\frac{F}{A_g p_c} + \frac{m \cdot M_x}{M_b} + 0 \leq 1.0$$

$$m = 0.57$$



<b>The Steel Construction Institute</b>  Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843 <b>CALCULATION SHEET</b>	Job No.	Sheet <b>14</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
<p>Effective length ~ for a fully braced frame, <math>k_3 = \infty</math></p> <p>Assume <math>\frac{L_E}{L} = 1.0</math> for simplicity</p> <p><math>\therefore L_E = 4.5\text{m.}</math></p> <p>From published tables <math>A_g \cdot p_c = 6270\text{ kN.}</math></p> <p>also for <math>L_E = 4.5\text{m, } M_b = 1100\text{ kN.m.}</math></p> <p>Check <math>\frac{3720}{6270} + \frac{0.57 \times 492}{1100} = \underline{0.85} &lt; 1.0</math></p> <p><math>\therefore</math> <u>Section Satisfactory.</u></p> <p>Design Case 3 ~ maximum axial load + associated moment.</p> <p>By inspection this case is not critical</p> <p>Use <u>305x305x198 UC</u> Grade 50.</p>			



<p><b>The Steel Construction Institute</b></p> <p>Silwood Park Ascot Berks SL5 7QN          Telephone: (0990) 23345          Fax: (0990) 22944 Telex: 846843</p> <p><b>CALCULATION SHEET</b></p>	Job No.	Sheet <b>15</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>J.M.</i>	Date <i>April, 89</i>	

Beam ~ Serviceability Condition

Non-Composite Moments/Stress  
 Unfactored load from secondary beam = 39 kN.

$$\therefore F.E.M = \frac{39 \times 18 (36-1)}{12 \times 6} + \frac{1.5 \times 18^2}{12} = 382 \text{ kN.m.}$$

$\phi_c = 2.06$ , (from sheet 5).

$$\therefore M_{\text{support}} = \frac{7 \times 2.06}{(7 \times 2.06 + 2)} \times 382 = 335 \text{ kN.m.}$$

Maximum free bending moment

$$= \frac{39 \times 6 \times 18}{8} + \frac{1.5 \times 18^2}{8} = 587 \text{ kN.m.}$$

Hence maximum positive moment

$$= 587 - 335 = 252 \text{ kN.m.}$$

maximum positive stress =  $\frac{252 \times 10^6}{3220 \times 10^3} = 78 \text{ N/mm}^2$

Composite section Properties


from sheet 6  $I_{bc} = 228000 \text{ cm}^4$   
 the elastic modulus of the steel tension flange is

$$Z_t = I_{bc} / (D + D_s - y_e)$$

$$= 228000 / \left( \frac{612 + 130 - 233}{10} \right) = 4479 \text{ cm}^3$$

for the concrete,  $Z_c = I_{bc} \cdot \alpha_e / y_e = 228000 \times 15 / 233 / 10$

$$= 146781 \text{ cm}^3$$

<b>The Steel Construction Institute</b>  Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843 <b>CALCULATION SHEET</b>	Job No.	Sheet <b>16</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
Date <i>April, 89</i>			

Composite Moments / Stress.

The unfactored superimposed dead load + imposed load from the secondary beams = 94 kN.

$$\therefore \text{F.E.M.} = \frac{94 \times 18 (36-1)}{12 \times 6} = 819 \text{ kN.m.}$$

$$\phi_c = 0.89, \text{ (see sheet 7).}$$

$$\therefore M_{\text{support}} = \frac{7 \times 0.89}{(7 \times 0.89 + 2)} \times 819 = 620 \text{ kN.m.}$$

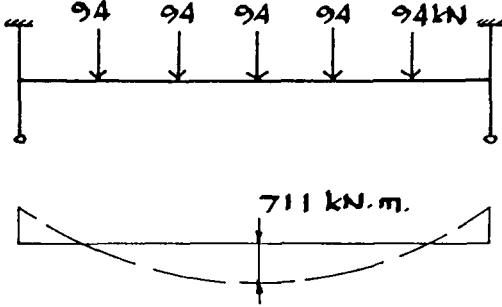
Reduce support moments by 10%

$$\therefore M_{\text{support}} = 0.9 \times 620 = 558 \text{ kN.m.}$$

Maximum free bending moment

$$= \frac{94 \times 6 \times 18}{8} = 1269 \text{ kN.m.}$$

Hence maximum positive moment

$$= 1269 - 558 = \underline{711 \text{ kN.m.}}$$



Maximum positive steel stress (bottom)

$$= \frac{711 \times 10^6}{4479 \times 10^3} = 159 \text{ N/mm}^2$$

Maximum concrete stress =  $\frac{711 \times 10^6}{146781 \times 10^3} = 48 \text{ N/mm}^2$

check  $0.5 f_{cu} = \underline{15 \text{ N/mm}^2} > \underline{48 \text{ N/mm}^2}$

$\therefore$  Concrete stress satisfactory.

<b>The Steel Construction Institute</b>  Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843 <b>CALCULATION SHEET</b>	Job No.	Sheet <b>17</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>J.M.</i>	Date <i>April, 89</i>	

Combined non-composite + composite steel stresses  
 $= 78 + 159 = 237 \text{ N/mm}^2$   
 Check  $p_y = 265 \text{ N/mm}^2 > 237 \text{ N/mm}^2$   
 $\therefore$  Steel stress Satisfactory.

Serviceability Deflections  
Imposed load deflections.

Unfactored imposed load per secondary beam = 81 kN.  
 $\therefore \text{FEM.} = \frac{81 \times 18 (36-1)}{12 \times 6} = 709 \text{ kN.m.}$   
 $\phi_c = 0.89,$   
 $\therefore M_{\text{support}} = \frac{7 \times 0.89}{7 \times 0.89 + 2} \times 709 = 537 \text{ kN.m.}$

reduce support moments by 10% (see Section 8.3)  
 $\therefore M_{\text{support}} = 0.9 \times 537 = 483 \text{ kN.m}$

Maximum free bending moment,  $M_0$   
 $= \frac{81 \times 6 \times 18}{8} = 1094 \text{ kN.m}$

for a symmetric load case, the mid span deflection can be found from~

$$\delta_{cc} = \delta'_c \left[ 1 - 0.6 (M_1 + M_2) / M_0 \right]$$


for full interaction, ie  $k=1.0$  for five equally spaced point loads~

$$\delta'_c = \frac{PL^3}{192EI} \cdot \eta \left[ 3 - \frac{1}{2} \left( 1 + \frac{4}{\eta^2} \right) \right] \quad \text{where } \eta = 6$$

$$= \frac{81 \times 18^3 \times 10^9 \times 6}{192 \times 205 \times 228000 \times 10^4} \left[ 3 - \frac{1}{2} \left( 1 + \frac{4}{36} \right) \right]$$

$$= \underline{77 \text{ mm.}}$$

For  $M_1 = M_2 = M_{\text{support}}$  it follows that:

<b>The Steel Construction Institute</b>  Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843 <b>CALCULATION SHEET</b>	Job No.	Sheet <b>18</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>[Signature]</i>	Date <i>April, 89</i>	

$$\delta_{cc} = 77 (1 - 0.6 (483 + 483) / 1094)$$

$$= 36 \text{ mm.}$$

Limit deflection to 50mm. or  $L/360 = \frac{18000}{360} = 50 \text{ mm.}$   
 Since 36mm < 50mm deflection satisfactory

Self weight deflection (non-composite)  
 Unfactored dead load from secondary beams = 39kN.  
 $M_{\text{support}} = 335 \text{ kN.m.}$   
 Maximum free bending moment,  $M_0 = 587 \text{ kN.m.}$   
 As before, the mid span deflection is given by ~

$$\delta_{cc} = \delta'_c (1 - 0.6 (M_1 + M_2) / M_0)$$


$$\delta'_c = \frac{39 \times 18^3 \times 10^9}{192 \times 205 \times 98600 \times 10^4} \times 6 \left[ 3 - \frac{1}{2} \left( 1 + \frac{4}{36} \right) \right]$$

$$+ \frac{5}{384} \times \frac{1.5 \times 18^4 \times 10^9}{205 \times 98600 \times 10^4} = 96 \text{ mm.}$$

$$\therefore \delta_{cc} = 96 (1 - 0.6 (335 + 335) / 587)$$

$$= 30 \text{ mm.} \quad (L/600)$$

No preambering required.

<b>The Steel Construction Institute</b>  Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843 <b>CALCULATION SHEET</b>	Job No.	Sheet <b>19</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>J.M.</i>	Date <i>April, 89</i>	

Dynamic Sensitivity

Combined floor frequency

main beam frequency,  $f_{main} = 18/\sqrt{\delta_{sw}}$  where  
 $\delta_{sw}$  = maximum deflection due to the inertia load,  
 ie. dead + 0.1 \* Imposed

The inertia load per secondary beam = 60kN

F.E.M. =  $\frac{60 \times 18(36-1)}{12 \times 6} + \frac{1.5 \times 18^2}{12} = 566 \text{ kN.m.}$

Maximum free moment

$$= \frac{60 \times 6 \times 18}{8} + \frac{1.5 \times 18^2}{8} = 871 \text{ kN.m.}$$

take  $\phi_c = 0.89$ , as before

$$\therefore M_{support} = \frac{7 \times 0.89}{(7 \times 0.89 + 2)} \times 566 = 428 \text{ kN.m.}$$

As before, the mid span deflection is given by ~

$$\delta_{cc} = \delta'_c (1 - 0.6(M_1 + M_2)/M_0)$$


$$\delta'_c = \frac{60 \times 18^3 \times 10^9}{192 \times 205 \times 228000 \times 10^4} \times 6 \left[ 3 - \frac{1}{2} \left( 1 + \frac{4}{36} \right) \right]$$

$$+ \frac{5}{384} \times \frac{1.5 \times 18^4 \times 10^9}{205 \times 228000 \times 10^4} = 61 \text{ mm.}$$

$$\therefore \delta_{sw} = \delta_{cc} = 61 \left( 1 - 0.6 \frac{(428 + 428)}{871} \right) = 25 \text{ mm.}$$

$$\therefore f_{main} = \frac{18}{\sqrt{25}} = 3.6 \text{ Hz}$$

Since 3.6 Hz > 3.0 Hz, beam satisfactory

<b>The Steel Construction Institute</b>  Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843 <b>CALCULATION SHEET</b>	Job No.	Sheet 20 of 31	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>Z.M.</i>	Date <i>April, 89</i>	

The natural frequency of the secondary beam was found to be  $\approx 17 \text{ Hz}$ . Hence the floor frequency is found from~

$$\frac{1}{f_0^2} = \frac{1}{f_{\text{main}}^2} + \frac{1}{f_{\text{secondary}}^2} = \frac{1}{3.6^2} + \frac{1}{17^2} \therefore f_0 = 3.5 \text{ Hz}$$

Since  $3.5 \text{ Hz} > 3.0 \text{ Hz}$  floor frequency acceptable

Dynamic Response Factor [see reference (21)]

for  $f_0 < 7 \text{ Hz}$ ,  $R = \frac{68000 C_f}{m \cdot S \cdot \text{Leff} \cdot \zeta}$

$m =$  floor mass ( $\text{kg./m}^2$ )

$$= \left[ \frac{60}{3 \times 6} + \frac{1.5}{6} \right] \frac{10^3}{9.81} = 365 \text{ kg./m}^2$$

the damping coefficient  $\zeta = 0.03$  for normally furnished floors referring to Table 7.1 case (3) of the vibration design guide, <sup>(21)</sup>  $S = W = 18.0 \text{ m}$ .

$\text{Leff} \sim$

Main beam relative flexibility

$$= \frac{1}{f_{\text{main}}^2} / \frac{1}{f_0^2} = \frac{1}{3.6^2} / \frac{1}{3.5^2} > 0.6$$

$\therefore \text{Leff} = L = 3.8 \left[ \frac{E I_b}{m \cdot b \cdot f_0^2} \right]^{1/4} \times L (= 6.0 \text{ m})$

$E I_b =$  dynamic flexural rigidity of composite secondary beam ( $\text{N. m}^2$ ).

$E I_b$  was found to be  $33.2 \times 10^6 \text{ N. m}^2$

$b =$  secondary beam spacing =  $3.0 \text{ m}$ .

<p><b>The Steel Construction Institute</b></p> <p>Silwood Park Ascot Berks SL5 7QN          Telephone: (0990) 23345          Fax: (0990) 22944 Telex: 846843</p> <p><b>CALCULATION SHEET</b></p>	Job No.	Sheet <b>21</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>J.M.</i>	Date <i>April, 89</i>	

$\therefore L^* = 3.8 \left[ \frac{33.2 \times 10^6}{365 \times 3 \times 3.5^2} \right]^{1/4} = 27m. > L_{max}, \text{ say}$

where  $L_{max}$  = building length in secondary beam direction.

$\therefore L_{eff} = 27m.$

Hence  $R = \frac{68000 \times 0.4}{365 \times 18 \times 27 \times 0.03} = 5.1$

The maximum value of R recommended for a general office = 8

Since  $5.1 < 8$  response factor satisfactory.

Lateral Stability

Construction Condition  
 negative moment region ~ check length between column and first secondary beam.

check  $\bar{M} \leq M_b$  where  $\bar{M} = m \cdot M_A$

Take  $m = 1.0$  and  $M_A =$  moment at haunch tip  
 haunch length = 1200mm.  
 haunch tip moment =  $580 - 191 \times 1.2 = 351 \text{ kN.m.}$

$\therefore \bar{M} = 1.0 \times 351 = 351 \text{ kN.m.}$

find  $\lambda_{LT} = n.u.v. \lambda$ , hence  $p_b$ , then  $M_b = S_{xx} \cdot p_b$

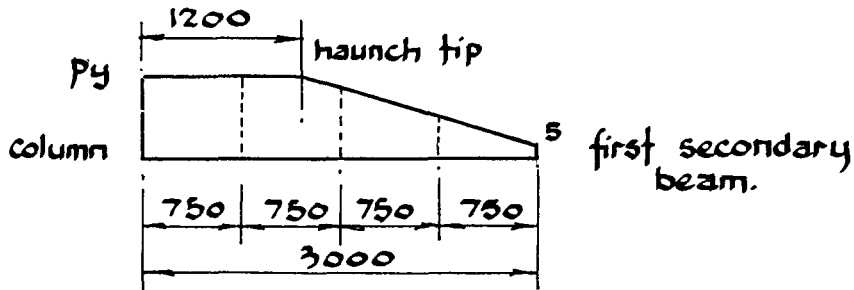
$n_t \sim$  Moment at 1st. secondary beam position

$= 580 - 191 \times 3.0 + \frac{1.4 \times 1.5 \times 3.0^2}{2} = 16 \text{ kN.m.}$

stress =  $\frac{16 \times 10^6}{3220 \times 10^3} = 5 \text{ N/mm}^2$



<b>The Steel Construction Institute</b> Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843  <b>CALCULATION SHEET</b>	Job No. _____ Sheet <b>22</b> of <b>31</b> Rev. _____	
	Job Title <b>APPENDIX A</b>	
	Subject <b>HAUNCHED BEAM DESIGN</b>	
	Client <b>SCI</b>	Made by <i>J.W.R.</i> Date <i>April, 89</i> Checked by <i>J.M.</i> Date <i>April, 89</i>



express  $N_1$  to  $N_5$  as applied stresses ~  
 $\therefore M_1$  to  $M_5$  becomes the yield stress  $P_y$ .  
 (this procedure is conservative).

then  $\frac{N_1}{M_1} = \frac{N_2}{M_2} = \frac{N_3}{M_3} = \frac{N_4}{M_4} = \frac{N_5}{M_5} = \frac{N_E}{M_E} = 1.0$  ,  $\frac{N_5}{M_5} = 0$

$\frac{N_3}{M_3} = 0.85$  ,  $\frac{N_4}{M_4} = 0.43$

$\therefore \eta_t = \left[ \frac{1}{12} \left\{ 1.0 + (3 \times 1) + (4 \times 0.85) + (3 \times 0.43) + 0 + 2(0) \right\} \right]^{1/2}$   
 $= 0.85$

$u = 0.873$

$v \sim w = \frac{I_{cf}}{I_{cf} + I_{tf}} = 0.5$  ,  $\lambda = L_E / r_y$

take  $L_E = 1.0 \times 3.0$   $\therefore \lambda = \frac{3000}{49.6} = 60.5$

Table 14  $\alpha = 34.0$  (BS.5950: Pt 1)

$\therefore \lambda / \alpha = \frac{60.5}{34.0} = 1.8$  ,  $\therefore v = 0.96$

$\lambda_{LT} = 0.85 \times 0.873 \times 0.96 \times 60.5 = 43.1$

Table 11  $p_b = 247 \text{ N/mm}^2$  (BS.5950: Pt 1)

$\therefore M_b = \frac{3680 \times 247}{10^3} = 908 \text{ kN.m} > 351 \text{ kN.m. OK}$

Negative moment stability satisfactory

<p><b>The Steel Construction Institute</b></p> <p>Silwood Park Ascot Berks SL5 7QN          Telephone: (0990) 23345          Fax: (0990) 22944 Telex: 846843</p> <p><b>CALCULATION SHEET</b></p>	Job No.	Sheet <b>23</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>J.M.</i>	Date <i>April, 89</i>	

Positive moment region  
 Check between the central pair of secondary beams.  
 for simplicity take  $m=1.0$  &  $\eta=1.0$   
 hence,  $\bar{M}$  = maximum span moment = 437 kN.m  
 and  $\lambda_{LT} = 1.0 \times 0.873 \times 0.96 \times 60.5 = 50.7$   
 Table 11  $\therefore p_b = 229 \text{ N/mm}^2$   
 $M_b = \frac{3680 \times 229}{10^3} = 843 \text{ kN.m.} > 437 \text{ kN.m.}$  Ok

Positive moment stability Satisfactory

Ultimate In-service Condition.  
 negative region ~ check length between column and first secondary beam.  
 find  $\lambda_{TB} = \eta_t \cdot u \cdot v \cdot c \cdot \lambda$  and check  $\frac{M}{S_x} \leq p_b$  i.e.  $M < M_b$   
 take  $\eta_t = 1.0$  and use properties at haunch tip.

$$v_t = \frac{1}{\left\{ 1 + \frac{1}{40} \left( \frac{\lambda}{\alpha} \right)^2 + \frac{1}{16} \left( \frac{L_n}{D} \right)^3 \frac{I_w \cdot L_n}{I_y} \right\}^{1/2}} \dots (18)$$

$L_n$  is lesser of  $L_{cr}$  or  $L_1$  = distance to secondary beam.  
 $L_{cr} = 3.74 I_y^{1/4} (D/t)^{3/4} \dots (19)$   
 $= 3.74 \times (3930 \times 10^4)^{1/4} (611.9/11.9)^{3/4} = 5686 \text{ mm.}$   
 $L_1 = 3000 \text{ mm.} < 5686 \text{ mm.} \therefore L_n = 3000 \text{ mm.}$   
 $\lambda = \frac{3000}{49.6} = 60.5, \alpha = 34.0 \therefore \lambda/\alpha = 1.8$   
 $I_w = \frac{t^3}{12} = \frac{11.9^3}{12} = 140 \text{ mm}^4, \frac{I_w \cdot L_n}{I_y} = \frac{140 \times 3000}{3930 \times 10^4} = 0.01$

The Steel Construction Institute



Silwood Park Ascot Berks SL5 7QN  
 Telephone: (0990) 23345  
 Fax: (0990) 22944 Telex: 846843

CALCULATION SHEET

Job No.	Sheet 24 of 31	Rev.
Job Title <b>APPENDIX A</b>		
Subject <b>HAUCHED BEAM DESIGN</b>		
Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>JAM</i>	Date <i>April, 89</i>

$$\therefore V_t = \frac{1}{\left\{1 + \frac{1}{40}(1.8)^2 + \frac{1}{16}\left(\frac{3000}{612}\right)^2 \times 0.01\right\}^{1/2}} = 0.93$$

take  $c=1 \therefore \lambda_{TB} = 1.0 \times 0.873 \times 0.93 \times 1.0 \times 60.5 = 49.1$

$\therefore p_b \approx 231 \text{ N/mm}^2, M_b = \frac{3680 \times 231}{10^3} = 850 \text{ kN.m.}$

$M =$  moment at haunch tip (see BM. diagram on sheet 11).

$= 758 \text{ kN.m.} < 850 \text{ kN.m.}$

$\therefore$  No additional restraint required.

Haunch Connection Design (Ultimate Load).

Design moment

Design moment = 1388 kN.m.

Try M27 bolts Grade 8.8

Tension capacity  $P_t = p_t \cdot A_t = \frac{450 \times 459}{10^3} = 207 \text{ kN.}$

Shear capacity  $P_s = p_s \cdot A_s = \frac{375 \times 459}{10^3} = 172 \text{ kN.}$

Minimum spacing down web,  $L_{min}$  is given by ~

$L_{min} \times 11.9 \times 265 = 2 \times 207 \times 10^3$

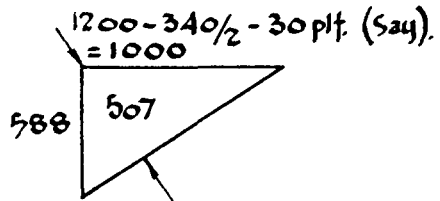
$\therefore L_{min} = 131 \text{ mm, but try } 135 \text{ mm.}$

Try 8 tension bolts and 4 shear bolts

cutting depth = 588 mm.

check cutting can be fabricated from same section.

Check 612-507-10 say  $> 20$  Flange thickness  
 $\therefore$  Adequate



check floor zone  
 $= 612 + 588 + 130 + 90 + 180$   
 $= 1600 \text{ mm.} \therefore \text{OK.}$

The Steel Construction Institute

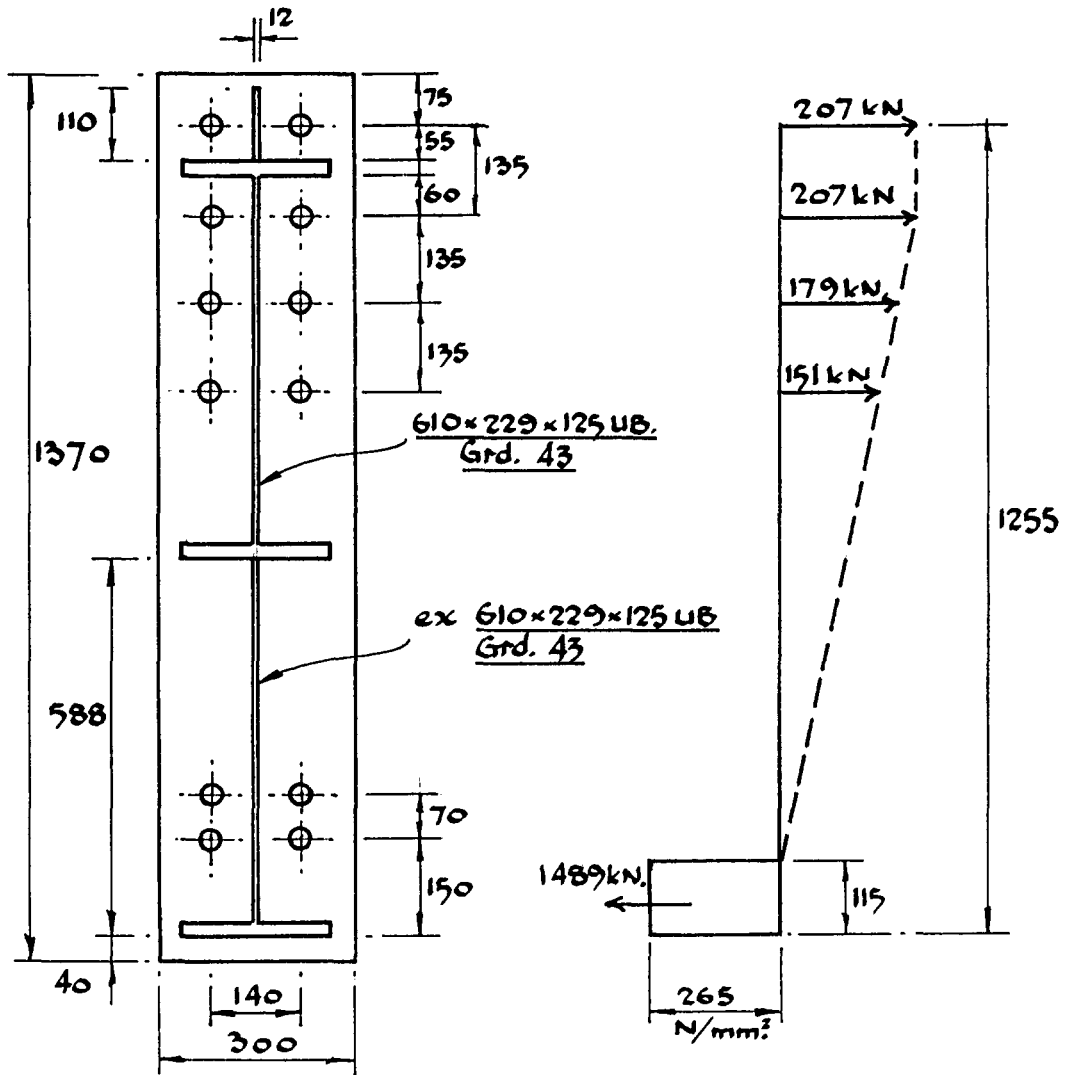


Silwood Park Ascot Berks SL5 7QN  
 Telephone: (0990) 23345  
 Fax: (0990) 22944 Telex: 846843

CALCULATION SHEET

Job No.	Sheet 25 of 31	Rev.
Job Title	APPENDIX A	
Subject	HAUNCHED BEAM DESIGN	
Client	Made by	Date
SCI	J.W.R.	April, 89
	Checked by	Date
	[Signature]	April, 89

Ultimate Moment Capacity  
Haunch Connection and Force Diagram.



Shear Capacity =  $4 \times 172 = 688 \text{ kN} > 526 \text{ kN}$ .

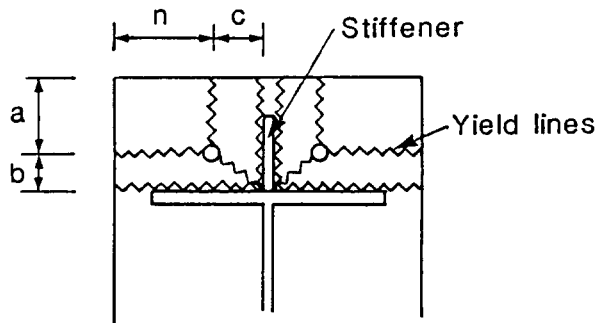
Moment Capacity =  $1562 \text{ kN.m} > 1388 \text{ kN.m}$ .

$\therefore$  Connection Adequate

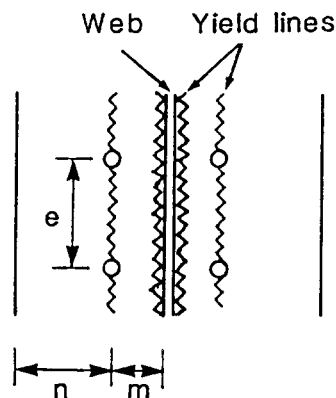
## Commentary to calculation sheet


### End Plate Design

The local yield line pattern in the end plate is as shown below. This is influenced by the presence of the stiffener. Double curvature is developed in the end plate which increases the bolt forces by prying action. The plate bending resistance multiplied by the factor representing the work done in deforming the plate along its assumed yield lines is compared to the applied force per bolt,  $F$ . If prying action is not included in the analysis, the plate moment resistance is taken as  $t^2/5$  (as suggested in Section 10.2 point 9).



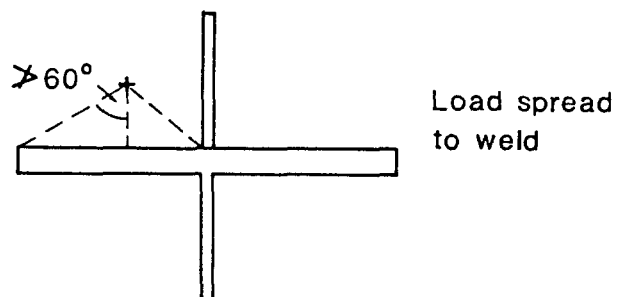
The design of the end plate is also influenced by the transfer of force to the beam web. The worst case is normally for the third row of bolts as there is no benefit from the adjacent flange. The plate moment resistance is obtained assuming double curvature is developed in the end plate as shown below. This results in a greater end plate thickness than the above case.



<b>The Steel Construction Institute</b>  Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843 <b>CALCULATION SHEET</b>	Job No.	Sheet <b>26</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
<p><u>End Plate</u>          Consider top 4 bolts ~ yield line pattern</p> <p> <math>a = 75 \text{ mm}, b = 55 \text{ mm}, c = 70 - 12/6 = 64 \text{ mm}.</math>          check <math>1.7b = 1.7 \times 55 = 94 \text{ mm} &gt; a</math>  <math>\eta = (300 - 140)/2 = 80 \text{ mm}.</math>          check <math>c + 1.4b + a = 64 + 77 + 75 = 216 \text{ mm} &gt; 0.5P_2</math> </p> <p>The plate moment of resistance, <math>m_p</math> is given from ~</p> $F \leq 2m_p \left( \frac{a+b}{c} + \frac{c+n}{b} \right)$ $= \left( \frac{75+55}{64} + \frac{64+80}{55} \right) 2m_p = 9.30 m_p.$ <p>Now <math>F = 207 \text{ kN}</math>, Try Grd. 50 plf. with modulus = <math>t^2/5</math></p> $\therefore t^2/5 \times 340 \times 9.3 > 207 \times 10^3$ $\therefore t \geq \sqrt{\frac{207 \times 10^3 \times 5}{340 \times 9.3}} \quad \text{ie. } t \geq \underline{18.1 \text{ mm.}}$ <p>Consider 3rd. row of bolts ~ yield line pattern</p> $F \leq \frac{2m_p e}{m} = \frac{2m_p \cdot 135}{64} = 4.2 m_p.$ <p><math>F = 179 \text{ kN}, \therefore t \geq \sqrt{\frac{179 \times 10^3 \times 5}{340 \times 4.2}} \quad \therefore t \geq \underline{25.0 \text{ mm.}}</math></p> <p><u><math>\therefore</math> Use 25mm. thick plate Grade 50.</u></p>			


## *Commentary to calculation sheet*

*The load spread from the bolt to the adjacent weld is taken as not greater than  $60^\circ$  as shown below.*



*The strength of the weld is taken as for an E51 electrode as in Table 36 of BS5950:Part 1.*



<p>The Steel Construction Institute</p>  <p>Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843</p> <p>CALCULATION SHEET</p>	Job No.	Sheet 27 of 31	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>[Signature]</i>	Date <i>April, 89</i>	

Beam to End Plate Welds.

Top flange welds~

Force in top 4 bolts =  $4 \times 207 = 828 \text{ kN}$

Try fillet weld, min. leg length =  $\frac{828 \times 10^3}{2 \times 227 \times 215 \times 0.7} = 12.0 \text{ mm.}$

Web welds~

bolt load spread length =  $2 \times 70 \tan 60^\circ = 242 \text{ mm.}$

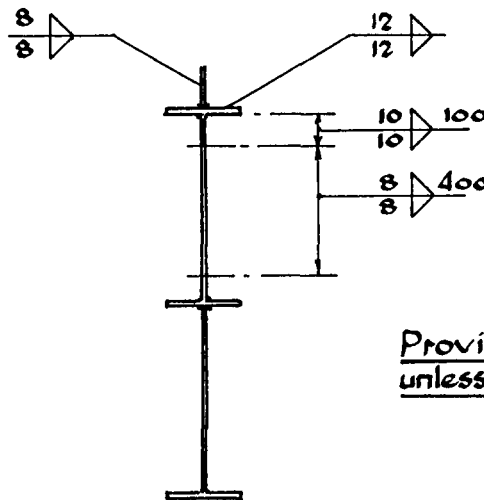
bolt spacing vertically = 135 mm. < 242 mm ∴ check on 135 mm.  
consider 3rd row, F = 179 kN.

∴ min. leg length =  $\frac{179 \times 10^3}{0.7 \times 135 \times 215} = 8.8 \text{ mm.}$

for the 4th row, F = 151 kN.

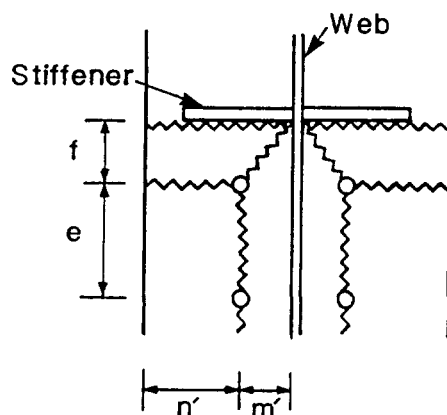
∴ min. leg length =  $\frac{151 \times 10^3}{0.7 \times 135 \times 215} = 7.4 \text{ mm.}$

by inspection provide weld as follows~




Provide 6mm fillet weld unless otherwise stated.

## Commentary to calculation sheet



*The column flange is subject to local bending via the bolts in tension. A 12mm stiffener is used to reduce this effect. The column flange is assumed to bend in double curvature and the yield line mechanism is as shown.*

*However, the column flanges are also subject to high axial stress which reduces the effectiveness of the horizontal yield lines. Their contribution is multiplied by a factor  $\mu$ . Because of the high axial stress in the flanges  $\mu$  is taken as 0. The applied force per bolt  $F$  is given in terms of the bending resistance of the column flanges times a factor representing the work done along the yield lines. Prying forces are not included in this analysis if the plastic section modulus is taken as  $t^2/5$ . (See section 10.2).*

<b>The Steel Construction Institute</b>  Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843 <b>CALCULATION SHEET</b>	Job No.	Sheet <b>28</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>J.M.</i>	Date <i>Apr 27, 89</i>	

Cutting Web to Beam Flange weld

50% of beam flange capacity =  $\frac{229 \times 19.6 \times 265}{2 \times 10^3} = 595 \text{ kN}$ .

Beam depth = 612 mm.  
Cutting length = 1000 mm. < 2 × 612 mm.

∴ design weld length = 2 × 612 = 1224 mm.

∴ min. leg length =  $\frac{595 \times 10^3}{1224 \times 215 \times 0.7} = 3.2 \text{ mm}$ .

Provide 5mm fillet weld.

Cutting toe weld

Weld length = 229 mm.  
50% beam flange capacity = 595 kN

∴ min. leg length =  $\frac{595 \times 10^3}{229 \times 215 \times 0.7} = 17.2 \text{ mm}$

∴ Provide 18mm fillet weld or butt weld

Column flange bending

Coexistent stress in flange,  $\sigma$   
consider column design case 1, just above 1st. floor ~

$$\sigma = \frac{714 \times 10^6}{2990 \times 10^3} + \frac{3119 \times 10^3}{252 \times 10^2} = \frac{363 \text{ N/mm}^2}{\text{over}} > \frac{340 \text{ N/mm}^2}{\text{over}}$$

hence interaction factor  $u = \left\{ 1 - \left( \frac{\sigma}{p_y} \right)^2 \right\} < 0$

∴ neglect horizontal components of yield lines  
ie. put  $u = 0$

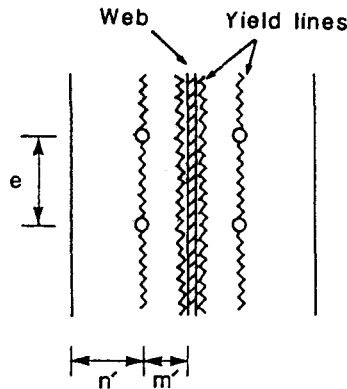
Consider 2nd. row of bolts ~ yield line pattern  
as commentary. Assume a 12mm. tension stiffener.

for  $u = 0$ ,  $F \leq 2m'p \left( \frac{f + 0.5e}{m'} \right)$

$f = 60 + \frac{(20 - 12)}{2} = 64 \text{ mm}$ ,  $e = 135 \text{ mm}$ ,  $F = 207 \text{ kN}$ .

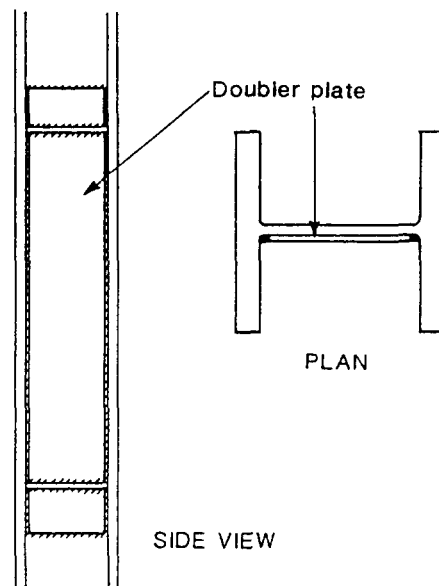
## Commentary to calculation sheet


Alternatively, the column flange is checked considering the yield lines adjacent to the third row of bolts as shown below. Again, the flange is in double curvature.



The column web panel in the region of the haunch is subject to high shear. In principle the effect of shear on moment capacity can be taken into account in Clause 4.2.6 of BS5950:Part 1. The local capacity of the column can then be rechecked, allowing for any reduction in moment capacity due to shear.

In some circumstances it may be necessary to increase the size of the column. Alternatively, a web doubler plate may be used, such that the net shear stress in the web does not exceed  $0.6P_v$  (so that there is no reduction in moment capacity). The shear force in the doubler plate may be taken as  $P_v$  provided the strength of the connecting welds is adequate. The doubler plate is welded to the web of the column before the stiffeners.



<b>The Steel Construction Institute</b>  Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843 <b>CALCULATION SHEET</b>	Job No.	Sheet <b>29</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>Z.M.</i>	Date <i>April, 89</i>	

$m' = 70 - \frac{19.2}{2} = 60 \text{ mm}, \quad m_p' = \frac{t^2}{5} \cdot p_y$

$\therefore t \geq \sqrt{\frac{207 \times 10^3 \times 5}{340} \cdot \frac{60}{2(64 + 0.5 \times 135)}} = \underline{26.4 \text{ mm.}}$

Consider 3rd. row of bolts ~

$F \leq \frac{2m_p' \cdot e}{m}, \quad F = 179 \text{ kN.}$

$\therefore t \geq \sqrt{\frac{179 \times 10^3 \times 5}{340} \cdot \frac{60}{2 \times 135}} = \underline{24.2 \text{ mm.}}$

since 26.4 mm and 24.2 mm < 31.4 mm

Flange Adequate

Check column web for shear ~

$F_v = 1489 \text{ kN.}$

$R_v = 0.6 \cdot p_y \cdot A_v = \frac{0.6 \times 340 \times 19.2 \times 339.9}{10^3} = 1331 \text{ kN.}$

since, 1489 kN > 1331 kN web inadequate

$\therefore$  Use doubler plate

limit force in web to  $0.6 R_v = 0.6 \times 1331 = 799 \text{ kN.}$


limit force in doubler plate to  $R_v$

$\therefore$  thickness is given by ~

$$\frac{0.6 \times 340 \times 339.9}{10^3} \times t = 1489 - 799$$

$\therefore t = \underline{10.0 \text{ mm.}}$

Provide 10mm. thick web doubler plate

<p>The Steel Construction Institute</p>  <p>Silwood Park Ascot Berks SL5 7QN Telephone: (0990) 23345 Fax: (0990) 22944 Telex: 846843</p> <p>CALCULATION SHEET</p>	Job No.	Sheet 30 of 31	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.G.R.</i>	Date <i>April, 89</i>
	Checked by <i>J.M.</i>	Date <i>April, 89</i>	

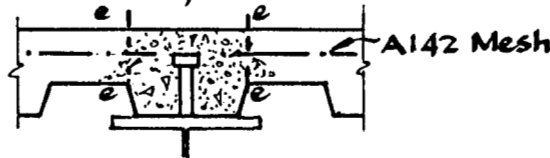
Longitudinal Shear (positive moment region only).

Total top flange longitudinal shear force per unit length

$$v = \frac{N Q_p}{s} = \frac{1 \times 72}{0.13} = 554 \text{ kN/m.}$$

Resistance of concrete flange ~

Potential shear planes for slab with decking parallel to main beam is shown thus



shear resistance per shear surface, e-e

$$V_r = 0.7 A_{sv} f_y + 0.03 \eta A_{cv} f_{cu} + V_p$$

but  $\nless 0.8 \eta A_{cv} \sqrt{f_{cu}} + V_p$

Take  $v_p = 0$ , ie ignore the contribution of decking since the position of lap joints is not known.

Hence for 2 shear planes, the shear resistance

$$= 2 (0.7 \times 142 \times 460 + 0.03 \times 0.8 \times 80 \times 10^3 \times 30) 10^{-3}$$

$$= 207 \text{ kN} < 554 \text{ kN.}$$

$\therefore$  Additional reinforcement required.

but check

$$2 (0.8 \times 0.8 \times 80 \times 10^3 \sqrt{30}) 10^{-3}$$

$$= 561 \text{ kN} > 207 \text{ kN. OK.}$$

<p><b>The Steel Construction Institute</b></p> <p>Silwood Park Ascot Berks SL5 7QN          Telephone: (0990) 23345          Fax: (0990) 22944 Telex: 846843</p> <p><b>CALCULATION SHEET</b></p>	Job No.	Sheet <b>31</b> of <b>31</b>	Rev.
	Job Title <b>APPENDIX A</b>		
	Subject <b>HAUNCHED BEAM DESIGN</b>		
	Client <b>SCI</b>	Made by <i>J.W.R.</i>	Date <i>April, 89</i>
	Checked by <i>ZSM</i>	Date <i>Apr. 7, 89</i>	

Area of additional reinforcement,  $A'_{sv}$

$$= (554 - 207) \times 10^3 = 2 \times 0.7 \times A'_{sv} \times 460$$

$$\therefore A'_{sv} = 539 \text{ mm}^2/\text{m}.$$

Provide T12 at 200 mm. (565 mm<sup>2</sup>).

Bar cut off ~  
shear force diagram.

The cut off length  $x$  is given by

$$\frac{3150/2}{554/2} = \frac{(3150 - x)/2}{207/2}$$

$$\therefore x = 987 + 12 \text{ dia. (Anchorage)}$$

Bar length =  $(987 + 12 \times 12) \times 2 = 2262 \text{ mm}.$

$\therefore$  Provide bars 2300 mm Long.